XX. *Inquiries concerning the Elementary Laws of Electricity. Second Series.* *By* W. Snow Harris, *F.R.S. &c.*

Received May 11,—Read June 16, 1836.

1. Having frequently had occasion to employ the balance of torsion invented by Coulombe, on the indications of which so many important deductions in electricity rest, I have endeavoured at various times to free it from certain defects of a mechanical kind, and render it more completely available to the purposes of an electrometer. Some of the difficulties experienced by the experimentalist in the use of this valuable instrument had already engaged the attention of our talented countryman the late Professor Robinson of Edinburgh, who proposed an ingenious method of checking the swinging of the needle, generally set in motion whenever we turn the micrometer or electrify the insulated balls. In the course of these inquiries I was led to the construction of a new species of balance; it may be termed, from the peculiar mechanical principle on which it depends, a bifile balance. The reactive force of this instrument is not derived from any principle of elasticity, as in the balance of torsion, but is altogether dependent on gravity; it seems generally available in experimental physics, is extremely well adapted to the measurement of small forces of repulsion, and to researches in electricity and magnetism, and is easily converted into a common torsion balance when required, free from the difficulties before alluded to. A description of this new instrument, together with an account of some further researches into the elementary laws of electricity, may, I hope, be acceptable to the Royal Society.

2. If a needle $m\,n$, fig. 1, (Plate XXVIII.) be suspended by two equal and similar vertical filaments of silk without torsion, $a\,b$, $a'\,b'$, placed parallel to each other at equal distances from the centre $c'\,c$, and fixed at the points $a\,a'$, it is evident that its position of rest will be horizontal, and in the vertical plane passing through the two threads. Whenever, therefore, we turn the needle from this position about the imaginary axis $c\,c'$, the lines of suspension will become deflected from the vertical, so that the distance $c\,c'$ will be shortened. We have hence a reactive force derived from the weight of the needle, which becomes imparted, as it were, to the threads of suspension; since the centre of gravity of the mass will again tend to rest in its previous position, and will be in a similar condition to that of a body falling down a very small circular arc. If therefore the needle be freely abandoned to this reactive force, a vibratory motion will arise, by observing which we may determine by the formulæ for oscillating bodies the nature of the reactive force producing the oscillations.

3. With this view a cylinder of wood, P, fig. 2, of two inches in altitude and two inches diameter, was suspended by two parallel filaments of unspun silk, $a\,b$, $a'\,b'$, fixed in the points $b\,b'$ in a diameter of the upper surface of the cylinder at equal distances from its centre c, an index i being attached to it, for the purpose of observing on a graduated card $i\,g$ the duration and extent of the oscillations. The threads $a\,b$, $a'\,b'$ were suspended in a convenient frame, $s\,d\,e\,s$, Plate XXIX. fig. 13, sustained by a firm base A B, elevated on levelling screws; each thread, after passing through fine holes in a moveable bar $r\,r'$, was joined above to a strong piece of silk line $a\,a'$, continued through holes in the fixed bar $d\,e$, being finally attached to regulating pegs at d and e. By this arrangement the respective lengths of the threads of oscillation $a\,b$, $a'\,b'$ were readily adjusted, so as to cause the cylindrical weight P to hang parallel with the plane of the card $i\,g$, fig. 2. By changing the situation of the bar $r\,r'$, fig. 13, the lengths of the threads of oscillation could be varied; and by a succession of fine holes in the bar $r\,r'$ corresponding to holes in the cylindrical weight P, it was also easy to vary their distance apart.

4. Having obtained in this way a given length and distance between the threads, the centre of the cylinder c, fig. 2, was caused to hang immediately over the centre of the graduated circle $i\,g$, as shown by contact with the finely-pointed extremity of a vertical rod, passed through friction-corks in the central block D, fig. 13. Small stays, $s'\,s''\,s'''$, &c., of light reed or cork being now inserted between the threads at given distances (sect. 8. *v.*), in order to prevent them from collapsing; the index i was turned to an angle of 60°, and the weight P allowed to oscillate; the fine central point being subsequently depressed from beneath the base B B', fig. 13.

Experiment A.—By carefully noting the rate of oscillation, the following results were immediately arrived at, viz.:

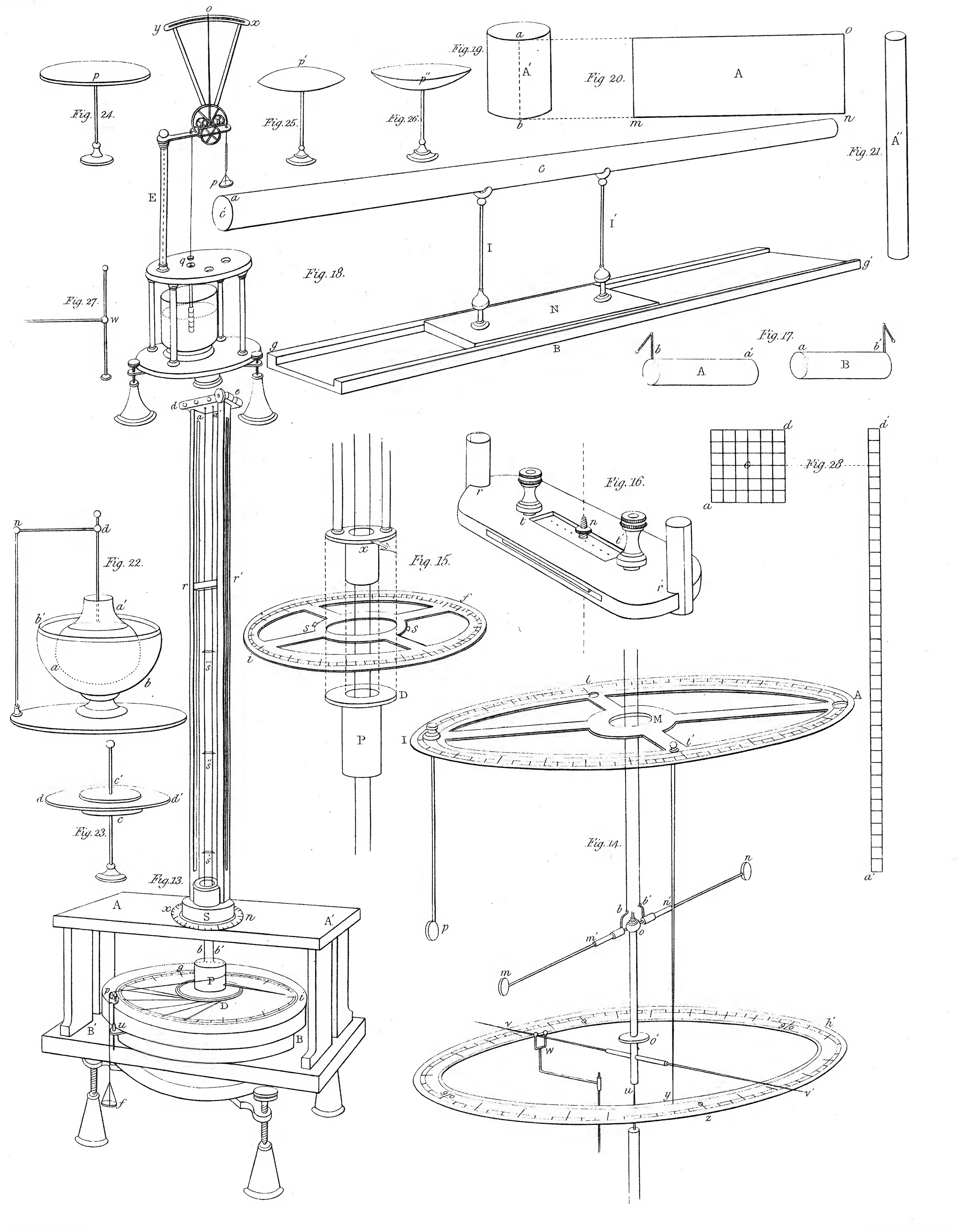
1°. The time of an oscillation is as the square root of the length of the threads of suspension divided by their distance apart, and is altogether independent of the weight of the oscillating body.

2°. The oscillations are isochronous at all angles.

5. From these results we may, by the general formula $n = \frac{P\,\pi^2\,a^2}{2\,g\,T^2}$, employed by Coulombe in his experiments on the torsion of wires, easily deduce the laws of the reactive force imparted to the threads.

In this formula, n is the force in terms of a unit of weight, = 1 grain, which, applied perpendicularly at the extremity of a lever of a unit of length, = 1 inch, will resist the reactive force imparted to the threads, when the cylinder c, figs. 2 and 13 has been turned about its axis, through an arc of 60°, whose chord is equal to the unit of length = radius. P is the weight of the cylinder c in terms of the unit of weight; a its radius; g the force of gravity, = 386 inches; T the time of an oscillation in seconds; π the ratio of the circumference to the diameter of the circle.

In applying this formula it is easy to perceive that the value of n will vary with



C. A. Goldsack, Delt.

Jas. Basire, Sculpt.

the squares of the distances between the threads of oscillation divided by their lengths, and that, contrary to the law of torsion as deduced by COULOMBE, is as the weight of the cylinder P: hence we have $n \propto \frac{P\,d^2}{l}$; and since the oscillations of the cylinder are found by experiment to be isochronous at all angles (4.), we may conclude that n is also proportional to the angle of deflection of the threads, as shown by the index P i, fig. 13.

6. These results were verified experimentally in the following manner:

Experiment B.—A weight f was caused to act tangentially to the circumference of the cylinder P, fig. 13, by means of a slender filament of silk, $f\,p$ P, passing over an extremely delicate pully p. This pully was attached to a slide and socket u, fixed to the circular rim $u\,g\,i$, moveable about the interior block carrying the graduated card $p\,g\,i$; hence the line $f\,p$ P could be exactly set at right angles to the radius of deflection in all positions of the index i, and the precise weight determined requisite to balance the reactive force of the threads at any given angle, or otherwise by turning the whole frame of suspension $d\,s\,e$ in a circular socket formed in the transverse piece A A′ through any number of degrees, as measured by a graduated card $n\,x$ and index x, and preserving always the index P i of the cylindrical weight at zero, we arrive at the reactive force of the threads of suspension, at any required angle, in a similar manner.

The results of a series of experiments conducted in this way completely verified the above deductions, the weight requisite to maintain the index at an angle of 60° being as the weight of the cylinder P multiplied into the squares of the distances between the filaments of suspension divided by their length. It was also found to vary accurately with the angle of deflection, the attendant circumstances being unchanged.

7. The following Tables, abridged from a greater number of experiments than it is desirable to mention here, afford a sufficient practical evidence of the truth of these results. In these Tables the unit of weight is 1 grain, the unit of length 1 inch, the unit of time 1 second.

TABLE I.

Showing the Rate of Oscillation with different Lengths and Distances of the Threads, as in Sect. 4.

Weight of cylinder = 960 grains. Angle of oscillation 45°.

Length.	Distance.	Oscillations in 60″.	Time of ten oscillations by observation.	Time of one oscillation.
6	0·25	28·50	21	2·1
24		14·25	42	4·2
6	0·4	46	13·1	1·31
12		32·50	18·5	1·85
24		23	26·2	2·62
24	0·8	46	13·1	1·31

Similar results were obtained when the angle of oscillation was increased to 180° and upwards, as also when the weight of the cylinder P was varied from 960 to 480 and 240 grains respectively, the radius being in each constant. The rate of oscillation was taken with a valuable chronometer belonging to my friend Colonel HAMILTON SMITH, and by which portions of time so little as the one sixtieth part of a second could be well estimated.

TABLE II.

Showing experimentally the Weight in Grains requisite to resist the reactive Force of the Threads at an Angle of 60°, their Length and Distance apart being varied, as also the Altitude of the Cylinder P (Exp. B.).

Length.	Distance.	Weight in grains on a lever of 1 inch.		
		P = 960 grains.	P = 480 grains.	P = 240 grains.
6	0·25	2·675	1·325	0·66
24		0·67 +	0·325	0·15
6	0·4	7 +	3·525	1·75
12		3·55	1·775	0·885
24		1·75	0·875	0·425
24	0·8	6·85	3·425	1·750

The smaller weights employed in these experiments could not be considered as mathematically exact: they were, however, sufficiently accurate for the purposes required. They consisted of 10ths of grains, 20ths, 40ths, and 100ths. The numerical values in the above Table are those which resulted from the position of the index, so far as these small weights could determine; and it will be seen that the approximation to the values deducible from the preceding Table I., by means of the formula $n = \frac{P \pi^2 a^2}{2 g l}$, are as near as could be expected from such an experiment.

TABLE III.

Showing the Weight in Grains, by Calculation and Experiment, required to balance the reactive Force of the Threads at various Angles of Deflection from 0 to 300°, the Threads being 24 inches in length and ·25 apart, and prevented from collapsing by seven small Stays, $s\ s'\ s''$, fig. 13, inserted between them at equal Distances (8.) (v.).

Weight of cylinder P = 960 grains.

Angle of deflection..	10	20	30	60	90	100	120	150	180	200	240	270	300
Force by formula ..	·115	·23	·34	·69	1·03	1·15	1·38	1·72	2·07	2·3	2·76	3·1	3·45
Force by experiment	·11+	·22	·34−	·67+	1	1·1	1·35	1·7	2	2·25	2·725	3 +	3·425

The pulley p, fig. 13, employed in these experiments was extremely delicate. The small scale f, in which the weights were placed, weighed the ·1 of a grain, and was suspended by a filament of the thread of a silkworm; that part of it passing over the pulley being particularly slender and flexible. The ·01 of a grain was by this means rendered very sensible on the index. The approximations in this Table are sufficiently close to show that the reactive force of the threads is as the angle of deflection (Exp. A.).

8. Upon these data the electrometer represented in fig. 3 has been contrived, which the following description will, I hope, render sufficiently intelligible.

α. Fig. 3, $a'' d$ is a cubical box or cage of about 14 inches square and 10 inches high; its vertical faces consist of large panes of glass loosely inserted in the grooved edges of light mahogany pillars about ·2 of an inch in diameter; $h\ c$, $d\ l$ represent two of the columns, and $d\ h$ one of the glass faces. The upper edges of these panes project above, so as to be easily removed when requisite.

The base of the cage, $d\ k$, consists of a clamped square of well-seasoned mahogany; the boundary $a''\ h\ l\ f$ of the upper part or roof is a stout framework of the same wood, through which the glass faces pass: the frame is firmly connected to the base by the intervention of the wood columns; each column is connected to the framework above by short brass shoulders fixed in their upper ends, and screwed into small plates of brass at the angular parts of the frame, as at the points $h\ l$; the horizontal grooves for the passage of the glass sides are continued through these plates in common with the woodwork of the frame; the base $k\ d$ is secured beneath by similar shoulders fixed in the lower ends of the columns: these last pass freely through the base at the lower angles $k\ c\ d$ into a flattened spherical nut, by which the whole is hove tight; each nut terminates in a cylindrical brass leg about an inch and a half in length and a quarter of an inch thick, as represented at $k'\ c'\ d'$. When the nuts are hove tight, the skeleton of the cage is very firm and complete: the cylindrical legs keep the instrument raised for about two inches above the table; they also serve to steady it on four pillars, as at $k'\ c'\ d'$, each five inches in length, and upon which the whole instrument is occasionally elevated for the purposes of experiment. The pillars last mentioned are united below to a clamped square of mahogany supported on levelling-screws, and have holes drilled into their upper ends for the reception of the cylindrical feet just mentioned.

β. An insulating needle $m\,n$, figs. 3 and 14, ten inches in length, is suspended within the cage by two vertical filaments of silk, $a\ b$, $a'\ b'$: it is connected with an index $v\,v'$, by means of a vertical rod $o\ u$, fixed to its centre. This index is about nine inches long, is set at right angles to the direction of the needle $m\ n$, and points out on a graduated circle, $v\ h'\ v'$, the angle of deflection from the point of rest. The graduated circle $v\ h'\ v'$, figs. 3 and 14, upon which the angular deflections of the needle are thus shown, is about nine inches in diameter: it is supported on four small pillars about an inch high, its centre being coincident with the central point at u.

γ. The needle $m\,n$, with its index, &c., is constructed in the following way: two light arms of glass, $m'\,n$, $n'\,m$, figs. 3 and 14, are inserted into the opposite ends of a small connecting joint, $m'\,n'$, about two inches or more in length, and ·2 of an inch in diameter, the whole forming one straight line. The connecting joint $m'\,n'$ consists of two pieces of light brass tube attached to opposite points of a small solid brass cylinder by short projecting studs: the central portion of the cylinder receives the extremity of the vertical rod $o\,u$. There are two sliders of brass on the connecting cylindrical tubes, which travel on them with a steady friction, and carry the hooks $b\,b'$ for the attachment of the suspension silks, which may be thus placed at any distance apart.

δ. A small circular area, n, of gold plate or light wood gilded, about ·4 of an inch in diameter and ·05 of an inch thick, with smoothed edges, is fixed to one extremity of the needle; and a small coated circle, m, of very thin varnished glass, or otherwise a similar disc, to the other. The coatings of the glass circle consist of circular pieces of gilt paper of about ·4 of an inch in diameter. This little element, when charged in the usual way with either electricity, maintains a very steady electrical state, not being liable to the variations which frequently arise in a simple insulated conductor; a property of great consequence in refined experiments, since in examining, from time to time, an insulated and similarly charged disc, p, figs. 3 and 14, or q and q', figs. 7 and 8, introduced within the cage, we thus obtain a more accurate measure of its electrical conditions than is usually arrived at by the electrization of a simple conductor. The small circular discs just mentioned are attached to the extremities of the needle $m\,n$ by a cement of shell lac, a portion of the glass rod near the end being ground away or drawn out in the flame of a lamp for a short distance so as to receive them; and thus the plane of the disc coincides with the axis of the rod. When the disc is of gilded wood, a small hole is drilled into its edge, or a corresponding flattened groove cut out of it for about two thirds of the radius, by which a similar result is obtained. The glass arms of the needle sustaining the discs are each covered with a thin coating of shell lac or good sealing-wax, the glass being previously cleaned, and heated sufficiently to melt the wax.

ε. The vertical brass rod $o\,u$, carrying the index $v\,v$, is about $4\frac{1}{2}$ inches long, and ·25 of an inch in diameter; its upper end terminates in a short shoulder; this shoulder passes through the centre of the connecting joint $m'\,n'$, and has a screw cut on it: there is a nut fitted to this screw, by which the two become completely united, and accurately set at right angles to each other.

ζ. The needle $m\,n$, with its index, &c., is suspended from the light frame $a\,x\,a'$, fig. 3: the suspension silks are connected by loops to the hooks $b\,b'$, and after passing through fine holes in the moveable plate $r\,r'$, and other holes in the fixed bar $a\,a'$ above it, are secured round the moveable pins $a\,a'$ for the convenience of adjustment.

η. The index $v\,v'$ is fixed horizontally to the rod $o\,u$, exactly at right angles to the

direction of the needle $m\ n$. It consists of pieces of light reeds of straw inserted one on each side in thin spring tubes; these are fixed on the ends of a short wire of brass about 2 inches in length, accurately passed through the rod $o\ u$, about $4\frac{1}{2}$ inches below the needle, so as to project from it for about $\frac{1}{2}$ an inch or more. A perfectly straight line may be obtained in this way by constructing each arm of two or more pieces of straw reed, fitted at their extremities one within the other, so as to present a sort of tapering appearance, the last piece being cut to a point, after the manner of a common writing-pen. Immediately over this index there is a sort of flattened stage of sheet brass, fitted to the rod by a spring tube, for receiving small circular weights, by which the reactive force imparted to the threads may be occasionally increased (6.). The weight of the needle, with its index, &c., is about 480 grains.

The lower extremity of the index-rod at u has a conical hole drilled up it, which allows the whole to play freely about a central pivot fixed in the end of a cylindrical rod $u\ z$. This rod passes through a spring socket immediately in the centre of the base of the cage, so as to be easily elevated or depressed by means of a milled head beneath, in which it terminates.

ϑ. A little on one side of the central point z, but very near it, two other brass rods, $e\ e'$, pass through the base in a similar way to the former; one of these, e, carries a bent rod $y\ w$, fixed on it by a small spring socket, and terminating in a sort of fork, w, about ·5 of an inch in width. One of the arms of the index $v\ v'$ may, by elevating or depressing the rod, be caused to pass through the fork, and thus the oscillations of the needle become completely checked in any point, or confined within a very limited number of degrees; the forked portions of the lever w are covered with some soft substance, so as to prevent, as much as possible, the index from rebounding. In this way by turning the rod e from beneath the base, the repulsive force acting on the needle is completely restrained, and gradually eased off to the point at which it becomes balanced by the reactive force of the threads. The opposite rod e' also carries a bent rod $e'\ w'$, by which we may at any time act on the other arm of the index, and thus obtain a very complete command over the oscillations of the needle. These two bent levers $w\ w'$ may be changed to either side, or one of them only may be employed, according as it suits the convenience of the experimentalist.

ι. Two other rods, $\pi\ \pi$, fig. 3, pass with friction through the base of the cage, in a line perpendicular to that of the rods $e'\ e$; these carry two small stages temporarily fixed on the extremities of the rods by spring sockets, to which they are attached. The milled heads of these rods are seen projecting under the base. The stages on elevating the rods support and bear up the needle $m\ n$, at the time of suspending it, or when the instrument is not in use, or otherwise when it is required to be moved about from one place to another. When the rods are elevated the connecting joint $m'\ n'$ of the needle is received between the perpendicular sides of the stages, as at $p\ p$, fig. 11; and thus it is at once supported and protected from damage, and its weight taken off the threads of suspension. The stages being temporarily fixed on the rods may be

removed, when required, at the time of using the instrument, and the ends of the rods depressed below the level of the base.

κ. The upper part or roof $a'' l$ of the cage $a'' d$ is, as already stated (α.), a wood frame, inclosing a circular opening of about a foot in diameter; this opening is just covered by the circle A t I, which is about 13 inches in diameter, and ·3 of an inch thick. The circumference of this circle is about an inch wide, and is divided upon its upper surface in each quadrant from 0 to 90°. It has two cross pieces or arms, $t\ t'$, A I, figs. 3 and 14, at right angles to each other; through the intersection of these there is a large round hole, M, fig. 14, for receiving the hollow vertical pivot, P, figs. 3 and 15, by which the circle may be accurately turned about its centre. The openings between the cross arms are covered in by thin glass plates, secured to the brass underneath by the heads of small screws. These screws project over the edges of the plates, the glass being defended by a thin leather collar. The under surface of the circle A t I is ground flat, and is smeared with a little grease of some kind, where it bears on the framework beneath, in order to allow of motion about the central pivot with an easy friction: the angular quantity by which this circle is at any time turned, is shown by a small bent index, I, outside the circumference, fixed either at I or A*.

λ. One of the cross bars A I carries the insulated disc p, or an insulated coated disc of glass g, fig. 5. These discs are similar to those of the needle already described. Each disc is insulated on a slender varnished glass rod, moveable in a spring socket, fixed in a plug of wood, as represented in fig. 5, so as to be readily adjusted to a given length. The plugs A or I, figs. 3 and 14, fit accurately in corresponding holes passing through the arms of the brass circle; they are placed in two similar and opposite points of the diameter A I, their centres being 10 inches apart, or a distance just equal to the length of the needle. By this arrangement either of the discs, or any other body, may be introduced within the cage of the instrument at pleasure, and the action on the needle observed.

μ. There is a vertical index-rod, $t'\ y$, figs. 3 and 14, in the diameter $t\ t'$ 90° distant from the holes A I, pointing immediately over zero of the graduated circle $v\ v'$ when the needle is in its position of rest, and the two discs $n\ p$ just touch. This index is sustained in a spring tube fixed in a neat wood plug, h, fig. 6, so as to be readily adjusted to any length, and can be applied at the opposite side t, if required.

ν. Beside the discs n, p, m, above mentioned, we may, by raising one of the glass faces, occasionally place within the cage any other electrified body whose condition we wish to examine, or otherwise small insulated proof planes, such as $q\ q'$, figs. 7 and 8. These are insulated on slender rods of varnished glass, $q\ h$, $h\ f$, fig. 7, set by an intervening ball of wood at right angles to each other. The horizontal rods vary from 2 to 5 inches in length; the vertical insulators are from 4 to 5 inches in length;

* The circle A t I may be of wood, having a graduated circle of card-board fixed on it; the remaining graduated circles may be also of stout card-board, where economy is an object.

these last are cemented below into a neat brass ferule, *f*, so as to be easily inserted when in use into a spring socket, *y*, fig. 7, attached to a narrow plate of brass or wood *y y'*. This plate is temporarily fixed in the base of the cage, as at *s*, fig. 3, in a line perpendicular to the direction of the needle when at rest. A portion of the interior of the plate is removed, as at *y y'*, and a screw, *s*, figs. 7 and 3, terminating in a milled head passed through it into a nut fixed in the wood underneath; the whole may be thus accurately adjusted so as to bring the proof plate *q* just in contact with the disc of the needle, when the index *v v* is at zero.

Either of the discs above mentioned may be connected with an insulated charged body, S, fig. 9, through a round hole H drilled in one of the glass sides of the cage: this hole is about ·6 of an inch in diameter, and is placed so as to admit of the external body being connected with the back of the disc; the communication is effected through the intervention of a light wire, which we may suppose to join *p* S, figs. 3 and 9. The wire is supported horizontally in a neat ball of wood attached to the extremity of a vertical rod of glass, placed in the base of the cage in the way already described (ν.), fig. 7.

One of the extremities of the connecting wire terminates in a fine point to be inserted in the back of the fixed disc *p* opposed to the disc of the needle; and the other extremity either in a small knob or a point, as the circumstances of the experiment require.

ξ. The brass circle A *t* I turns freely about its centre under a light transverse piece of brass, B B', about 4 inches in width at its extremities, and ·3 of an inch in thickness: this piece is fixed on short vertical pillars *i i'*, &c., so as to be raised above the roof of the cage, the whole being secured by screws and nuts.

ο. The transverse piece B B' is accurately centered, and a circular opening drilled through the centre at *x* of about an inch or more in diameter; for the sake of lightness it is formed as represented in the figure. Immediately in the centre of this piece there is united to it a short plate of brass, D, fig. 15, three inches in diameter and three tenths of an inch thick. This plate has a drawn brass tube, P, of an inch bore, fixed underneath it, which, passing centrally through the frame B B', retains the circle A *t* I, figs. 3 and 14, in its place beneath. There is a similar plate and descending tube, *x*, figs. 3 and 15, placed immediately over this, carrying the frame *a x a'* for the suspension threads of the needle. The hollow tubes fit closely one within the other, as indicated in fig. 15; and the faces of contact of the two plates are ground fair together, so as to admit of the upper plate being turned upon the lower one without materially deranging its vertical position.

π. In order to estimate the angular quantity which the upper plate has been turned, there is a graduated circle, *i f*, figs. 3 and 15, of six inches in diameter, sustained without the edge of the fixed plate D by light arms projecting from an interior ring. This ring is accurately fitted to the circular edge of the fixed plate within, so as to admit of its turning upon it as a centre for a short distance, with some friction; by

this the zero of the circle can be nicely set to any given point. The whole is then finally fixed in the required position by two small screws at $s\ s'$, fig. 15, passing through the interior ring against the edge of the plate within.

ϱ. Each quadrant of the graduated circle $i\ f$ is divided from 0 to 90°; and there is an index x projecting from the moveable plate above immediately over the divisions, indicating the angular quantity it has been turned, and consequently the amount of the deflective force impressed upon the needle $m\ n$ through its threads of suspension.

This arrangement, especially represented in fig. 15, enables us to retain the brass circle A I t and the frame $a\ x\ a'$ in their respective situations, without interfering with the required circular motions or the opening into the cage below for the threads of suspension (ζ.).

σ. The frame $a\ x\ a'$ may vary from one foot to three feet, according to the views of the experimentalist; it is slightly but firmly constructed of two light brass tubes, $x\ a$, $x\ a'$; these are received upon two short vertical rods, screwed into the plate x, and are united by a cross piece $a\ a'$ above. The extremities of this cross piece carry spring sockets for the two adjusting pegs $a\ a'$ already mentioned.

τ. The brass plate for regulating the length and distance of the suspension threads (β.) is steadied between two parallel pieces $r\ r'$ by means of small screws which tie them together; sufficient space being allowed for placing it accurately in the centre. The compressing pieces are united at each end by cross bars, and are soldered to short tubes which slide with friction on the vertical rods $x\ a$, $x\ a'$, as shown in fig. 16.

υ. The small stays $s\ s'$, fig. 3, inserted at given distances between the suspension threads, are formed and applied in the following manner. A short piece of finely grained cork, $g\ g'$, fig. 10, of the required length, about ·2 of an inch wide and ·1 of an inch in thickness, is cut nearly through at each extremity, and a horizontal portion $g\ a$, $g'\ a'$, removed, leaving a raised portion $a'\ a$ in the centre, exactly equal to the given distance between the threads. The filaments of suspension being each passed into the eye of a fine needle, are lightly smeared with a little oil, and run through the cork in the points of intersection of the depressed portion $g\ a$, $g'\ a'$, and the central straight line parallel to its sides. The stays will now hang with sufficient friction between the threads, and may be placed in the points required.

φ. In order to facilitate any required change in the sensibility of the instrument by varying the distance between the threads of suspension (5.), several sets are prepared at different distances apart, terminating above and below in small loops, as represented in fig. 12. The upper ends being passed into the eye of a fine needle, are easily run upwards through the holes in the plate $r\ r'$, fig. 3, at the given distances, and hooked on to the stout threads $a\ a'$ above, and to the hooks $b\ b'$ of the needle below.

9. When we wish to employ this instrument as a balance of torsion (1.), the needle, with its index, &c., is suspended by a metallic wire in the following way:—there is a small shoulder at the extremity of the rod $u\ o$, figs. 3 and 4, through which a small

hole is drilled, passing obliquely from the centre out at the side just below. The wire is passed through this hole, and its end finally secured by a small brass spherule *s*, fig. 4, accurately fitted to the end of the shoulder. When the brass spherule is gently forced into its situation, the wire is compressed, and becomes firmly united to the needle. Being thus secured below, it is continued through a fine central hole in the moveable plate $r\,r'$, fig. 3, and finally attached to an adjusting cylindrical peg at a'. It is completely stopped at any given length by a small conical screw and nut *n* fixed in the plate, fig. 16, and through which the wire passes; by tightening the nut the wire is compressed, and cannot therefore twist beyond a given point.

In order to balance the needle accurately, should it be required, the two sliders $b\,b'$, fig. 4, are turned so as to bring the suspension hooks underneath; this enables us to append to the hooks two small weights $w\,w'$, and place them at such respective distances from the centre as will accurately balance the needle $m\,n$ in a horizontal position*.

The instrument thus prepared becomes, by means of the micrometer at *x*, a very complete torsion balance, possessing many important advantages; amongst these is the means of increasing or diminishing its susceptibility by changing the length of the wire, Coulombe having shown that the reactive force of a wire is in a simple inverse ratio of its length.

10. The reactive force obtained in the way just shown, by means of two parallel filaments of silk, is more perfect, in a great variety of instances, than the elastic force of torsion, and is besides very manageable. The deflection of the threads from the vertical may at all times be extremely little; and the angular deviation of the needle seldom greatly exceeds 90°, although by increasing the number of stays $s'\,s''$, &c., fig. 12, it may with safety be carried through the whole circle (7.); for many refined inquiries, however, the angle of deflection need not exceed 30°. We must, however, remark, that in this, as in the case of the torsion balance, an error may arise in large arcs by taking the arc itself as the measure of the distance between the opposed bodies $p\,n$, fig. 3, and the length of the lever at the extremity of which the force acts, as equal to radius or half the length of the needle: these errors, however, in the instrument we have been describing, sufficiently balance each other, or very nearly so; one of the factors of the moment of the force being the cosine of half the angle of deflection, and consequently less than radius, or the lever upon which the repulsion operates, whilst the arc taken to measure the distance is always greater than its chord, or the real distance.

11. It is easy to perceive that the reactive force imparted to the threads of suspension may be varied in any proportion, either by changing the position of the sliding bar $r\,r'$, by which their length is altered, or otherwise by varying their distance apart (φ.); or finally, by the due adjustment of small circular weights placed on the

* A separate needle may be employed for this purpose, if desired.

small stage at o'; we may by either of these methods, taken separately or conjointly, vary the reactive power between very wide limits. In the instrument above mentioned a force so small as the $\frac{1}{20,000}$dth part of a grain for each degree is readily attained, and it may be diminished to the $\frac{1}{50,000}$dth.

12. The construction of this instrument having been completely described, a brief experimental explanation will suffice to make its practical application to the purposes of an electrometer easy and intelligible.

Let it, for example, be required to investigate the laws of the repulsive force exerted between the insulated bodies *p m*, figs. 3 and 14, under the variable conditions of distance, intensity of charge, and the like.

The instrument being elevated on its vertical pillars, as represented in fig. 3, is first levelled, and the central rod *o u* made, by a due adjustment of the plate *t t'*, fig. 16, to hang freely over the vertical pivot at *u*, at whatever angle the frame *a x a'* may be turned: the disc *m* of the needle is now adjusted, so as just to touch the fixed disc *p* in every point, or nearly so, the forked lever *w* being employed to retain the index *v v'* at zero. The fixed indexes I and *t' y* are then also set at zero of their circles A *t i* and *v h v'*, the forked lever *w* arresting, without undue pressure, the discs *m* and *p* in contact, when the index *v v'* is at zero. We now proceed to electrify the discs by means of any insulated charged body introduced within the cage, either through the circular hole H or under one of the glass sides, which can be raised for the purpose; this done, we turn the circle A *t* I through any given number of degrees in a direction *contrary* to that in which the needle would be repelled, the forked lever *w*, figs. 3 and 14, being gently eased away until the repulsive force becomes exactly balanced. In this case, as is evident, the distance between the repelling surfaces is measured by the number of degrees of the graduated circle *v h' v'*, intercepted between the vertical and horizontal indexes *t' y*, *v v'*, and the force itself by the number of degrees of the same circle contained between the final position of the index *v v'* and zero. Suppose, for example, the repelling discs were actually circumstanced as in fig. 14, then their distance apart is expressed by the arc *y v'*, and the force, by the arc *z v'*, the point *z* being zero of the card. If the point *y* coincided with *z*, then both the distance and force would be expressed by the arc *z v'*. If the index *t' y* be not employed, we estimate the distance of the bodies by means of the angular quantity indicated on the brass circle A *t* I, which must be in this case added to the angular deflection of the needle from zero. Let it now be required to examine the repulsive force at any other distance less than the former. We have only to move the circle A *t* I in the same direction as that in which the needle is repelled, and we immediately change the relation between the arcs *y v'* and *z v'*, the receding of the needle being at the same time gently checked by the fork of the lever *w*. We thus eventually obtain a new distance and force, measured in terms of the same graduated circle.

In this kind of inquiry we may, as it is evident, obtain deflections of the needle

from less than a single degree up to 360 degrees if requisite (7.) (Table III.), by turning the circle A t I either in a direction contrary to that in which the needle is repulsed, or otherwise in the same direction. We may in either case always estimate the distance of the repelling bodies independently of the vertical index $t\,y$, by adding or subtracting the angular turning of the circle A t I, to or from the angular deflection of the needle, according to the direction in which it has been moved.

If the disc m be connected with any external body, s, figs. 3 and 9, then the relative distances and forces may be varied by turning the micrometer index x a given number of degrees, either in the same or in a direction contrary to that in which the needle is repelled, which must be either added to, or subtracted from the total angular deviation of the index $v\,v'$, according as the direction is $+$ or $-$ in respect of the direction of the force of repulsion. Thus if, as in fig. 14, the deviation of the needle as expressed by the index $v\,v'$ amounted at first to 20°, and by subsequently turning the index x, figs. 3 and 15, by a quantity equal to 70°, in a direction contrary to that in which the needle was repelled, we had brought the index $v\,v'$ back to a distance of 10°, then the total force at 10° would be expressed by $10° + 70°$, that is, 80°. If, on the contrary, we had turned the micrometer index x 30° in the same direction as that of the repulsion, and had by this advanced the index, say to a distance of 40°, then the total force at 40° would be expressed by $40° - 30°$, that is, 10° only.

With respect to the distances of the electrified discs $p\,m$, they are always measured with sufficient accuracy by the arc contained between the vertical index $t\,y$ and horizontal index $v\,v'$, or by adding or subtracting the angular turning of the circle A t I.

13. There are one or two circumstances connected with the investigation of the laws of electrical action by means of this instrument, or the balance of torsion, requiring some attention. 1°. If any considerable time be employed in a particular experiment, the charged bodies will, under ordinary circumstances, lose some portion of their electricity. Coulombe endeavoured to avoid the error necessarily arising from this, by computing the amount of the electrical dissipation and applying a correction, and by some other methods incidental to the nature of the given inquiry. This process, however, is not always easy, or even safe; the dissipation of a charge being frequently a most uncertain operation; and a little more or a little less allowance for the assumed decrease of the charge makes an astonishing difference in delicate researches, the force by which a body loses its charge being as the square of the quantity of electricity. The only sure protection from this source of fallacy is to avoid it altogether, which can be always effected if we choose a proper season for experiment, and operate in a dry room moderately warmed by a common drying stove. It is surprising under these circumstances to see how completely the repelling bodies retain the charge; this, together with the facility of manipulating obtained by the instrument above mentioned, will be found to reduce indefinitely any error arising from dissipation. 2°. Coulombe found that some insulators did not insulate sufficiently, and were liable to become electrified; in consequence of this

his results were sometimes uncertain. We may also avoid this by exposing the insulators to the influence of small irons heated beyond redness, and prepared of various forms for the occasion, which at once deprives them of any electricity they may have taken up, and renders their insulating power sufficiently perfect. I have found slender glass rods covered with a varnish of coarse shell lac laid on warm, or otherwise good sealing-wax, become extremely perfect insulators when treated in this way.

14. The relation of the repulsive force between two bodies, as *p, m,* figs. 3 and 14, to the quantity of electricity with which each is charged, and to their distance apart, is of vital consequence to electrical investigations depending on the principle of repulsion. According to the experiments of M. COULOMBE, the total force between two insulated charged bodies, is as the quantity of electricity in each directly, and as the squares of the distances inversely*. Hence he supposes the action of each of the repelling bodies to enter into the composition of the result in such way, that the total force increases or diminishes with the electricity contained in either.

Thus the total force with which two bodies considered as points repel each other at any distance, D, is represented by $\frac{F}{D^2}$, or considering F as the product of two constants R R′, that is to say, of the force of each body, this expression becomes $\frac{R\,R'}{D^2}$. BIOT terms these constants R R′ the electrical reaction of the bodies to which they apply. Although this expression coincides in many cases with the results of experiment, it does not seem upon the whole sufficiently general. Having found many important exceptions to it, I was led to institute a series of experiments with a view of discovering the cases in which it might possibly fail.

Experiment C.—The results given in the following Table were deduced by repeatedly examining the repulsive force exerted between the electrified discs *p, m,* fig. 3, placed in a perfectly insulating atmosphere (13.) at various distances apart, and charged with equal and unequal quantities of electricity. The relative charges were obtained by the simple method resorted to by COULOMBE, viz. by touching one of the bodies with an insulated neutral body of the same dimensions, so as to abstract at any time one half the charge. In these experiments the quantity requisite to produce a repulsive force of 24° when equally divided between the two discs was taken as a unit of charge; the reactive force of the instrument being about the $\frac{1}{1500}$dth of a grain for each degree. I then proceeded to reduce the charge upon one of the discs in such given proportions as could be obtained by continually abstracting one half the quantity remaining on either disc. In deducing the results each disc was in turn made constant, and a mean result taken; this did not, however, upon the whole, greatly differ from that arrived at by one disc alone. The length of the threads of suspension was taken at 20 inches, and their distance apart ·25 of an inch,

* HAÜY's Philosophy, by GREGORY, p. 356 and 364. † BIOT, Traité de Physique, tom. ii. p. 242.

the reactive force at 60°, being about the $\frac{1}{25}$th of a grain, that is, about the $\frac{1}{1500}$dth of a grain for each degree, as already mentioned.

TABLE IV.

Distances.		Forces with charges equally reduced.			Forces with charges reduced and unequal.					
Degrees.	Ratio of distance.	1 : 1	$\frac{1}{2}:\frac{1}{2}$	$\frac{1}{4}:\frac{1}{4}$	$1:\frac{1}{2}$	$1:\frac{1}{4}$	$1:\frac{1}{8}$	$\frac{1}{2}:\frac{1}{4}$	$\frac{1}{2}:\frac{1}{8}$	$\frac{1}{4}:\frac{1}{8}$
2	$\frac{1}{12}$	....	....	46	....	Attracted.	Attracted.	....	Attracted.	
3	$\frac{1}{8}$	....	....	36	....	62	30	55	29	16
6	$\frac{1}{4}$	....	85	18	94	46	22	30	16	8
9	$\frac{3}{8}$	125	42	8+	70	34	16−	14	8	4·5
12	$\frac{1}{2}$	94	23	4·5	47	23	10+	8+	5−	2+
18	$\frac{3}{4}$	44	10·5	2+	20	10	5	3+	2	....
24	1	24	6	1+	12	6	3	2	1+	....
48	2	6	1·5	....	3	1·5	....	....	....	....
72	3	3	....	....	1+	....	....	....	....	....
D	R	A	B	C	*a*	*b*	*c*	*d*	*e*	*f*

15. The above Table exhibits the following phenomena, which are not a little striking and important.

1st. It may be perceived, that in columns A and B, the discs being charged equally and to a given intensity, the forces vary, (with one or two exceptions only,) in an inverse ratio of the squares of the respective distances: in column A there is only one exception, viz. at the distance 9°. When, however, we begin to diminish the quantity on one of the discs, or charge them unequally, this law is only apparent up to a certain limit. Thus at the distance 12° and 6° of columns *a*, *b*, and *c*, as also at 12° and 9° of column A, 6° and 9° of column C, the law is in an inverse ratio of the simple distance, or nearly approaching to it, whilst within certain limits, and at other distances, the law of the force becomes irregular, and is apparently disturbed by some foreign influence. Similar results are more or less apparent in all the other columns where the quantities of electricity in the repelling bodies are unequal.

2nd. It is observable that the deviations from the law of $\frac{1}{d^2}$ are more apparent and decided, when the forces are more diminished, the inequality of the respective charges greater, and the distance less: under any or all of these conditions, the rate of increase of the repulsive forces diminishes, and the repulsion verges toward, and is at length superseded by, attraction, as may be seen by a slight inspection of the Table.

3rd. The quantities of electricity contained in either of the repelling bodies are not always proportional to the repulsive forces. Thus in columns B and *d* the respective quantities on one of the bodies are as 2 : 1, the quantity on the other remaining the same, $= \frac{1}{2}$; but the respective repulsive forces are nearly as 3 : 1, or at least approximate very closely to this ratio. The same thing is, in some instances, apparent in columns *b* and *d*, where the quantities are in the proportions of $1:\frac{1}{4}$, and $\frac{1}{2}:\frac{1}{4}$. We

likewise observe in columns B and C that the respective quantities are as 2 : 1 on each of the discs; but the corresponding forces are not as 4 : 1, but nearly as 5 : 1 throughout.

16. Although these results may seem at first anomalous and unsatisfactory, they are still such as would be likely to arise out of the peculiar nature of electrical action, and may I believe, on due reflection, be found in complete accordance with its general laws. It may be shown, for example, that the force of induction in electricity is not confined to a charged and neutral body, but operates more or less freely, even between bodies *similarly* charged. Now whatever be the precise nature of the inductive process, it is present in every species of electrical action, although under certain circumstances its tendency to attraction may not be apparent, or be of a negative character.

Experiment D.—If, for example, two charged cylindrical conductors, A and B, fig. 17, have electroscopes, b b', connected with their distant ends, then, on approximating their near extremities a a', we shall observe a continual increase of force in the opposite ends b b' which will continue up to a certain limit according to the quantity of electricity with which the conductors are charged, and whether equally or unequally.

Now the inductive force of these cylinders on each other, as here evinced, may (in accordance with the doctrine of a single fluid, and merely *pour fixer les idées*,) be supposed to repel the electricity resident at their near extremities a' a, and cause it to retire towards the distant ends b' b. If the conductors A and B at the instant of this process be free to move, then the resistance to the increased accumulation towards the extremities b b' is attended by a mutual recession of the bodies, and they seem to repel each other. We do not however, under any circumstances, obtain really the whole repulsive force of which the quantities of electricity at first collected in the near extremities a a' would seem to be susceptible, since the operation of the inductive influence may be associated with a displacement of some of the agency on which the repulsion depends, and by which the quantity in the repelling ends is more or less diminished. If, however, the bodies be equally and highly charged, there may, in certain cases, be so little displacement at some distances, in comparison with the whole quantity accumulated, that the decrease of the force from this cause is of no great value. Supposing, however, the charges unequal, then the resistance in one body is greatly decreased, and there may arise so great a change in its repelling extremity, by the inductive influence, as to cause a very sensible diminution of the comparative repulsive force exerted at a given distance between the two bodies. If the disproportion of the respective charges be very great, then not only could all the free electricity of one of them, A, become displaced in its proximate extremity a', and so be actually reduced to a state of neutrality, but a further induction may arise, such as always occurs in the case of any other approximated neutral and charged surface, and which, although not so perfect as under the ordinary circumstances, owing to the

accumulation of the free electricity toward the opposite extremity *b*, may still generate a very sensible attractive force; the tendency of the inductive process being, first to raise the anti-attractive state of the bodies to zero; secondly, to generate in them an actual attractive power. There is consequently no essential distinction in this action, whether it take place between two bodies each similarly charged, or between a charged and neutral body, or even between two bodies dissimilarly charged; the only difference being, that in the latter case the inductions commence at a point already beyond a limit, which may be called zero; in the two former they commence in the one case at zero, in the other at some point below it. I hope to lay before the Royal Society, at no very distant period, some new researches on electrical induction and attraction, which may possibly throw some further light on the nature of electrical forces. It is quite evident that the inductive process between two bodies similarly charged may become indefinitely modified by the various circumstances of quantity, intensity, distance of the charged bodies, and the like; giving rise to apparently complicated phenomena, as I think appear in the results of the experiments just given in Table IV. Thus in column A, where the charges on the discs are equal and of a given intensity, the resistance in one body to the inductive tendency of the other is at the given distances so great, that little or no attractive effect ensues, and the repulsive force proceeds according to a certain law; but in columns C, *b*, *c*, *d*, &c., where the resistance to electrical change by induction in one or both of the bodies is less, the decrease of the force is more sensible, and as the distances diminish, becomes more and more evident, until at last the repulsive force no longer increases, and is in some cases superseded by attraction. It is only, therefore, within certain limits that we should expect to find the results of experiment, in any instance, conformable to a regular law, even although the bodies be equally charged: thus in column B it may be perceived that the law of the force has become irregular, and as the distances further diminish, varies inversely as the simple distance. Such is also the case in columns C, *c*.

17. *Experiment* E.—This conclusion was further verified by operating at first with charges of a low intensity at small distances, viz. between 0 and 10°, and then by greatly increasing the sensibility of the instrument, examining the action of the same charges at distances more extended, or otherwise by taking such electrical intensities as showed, with a given reactive force, the march of the repulsion between very extensive limits, the discs being either equally or unequally charged. The results were in strict accordance with those already given in Table IV.: the law of the force, which at first was as $\frac{1}{d^2}$, became at a certain point irregular as the distances decreased, and after being as $\frac{1}{d}$ became in some cases again irregular, until at last the repulsion vanished altogether, and was superseded by attraction.

18. The deviations from a uniform law of action being thus evident, even with two

insulated and similar bodies equally charged, we may conclude that they always occur, to a greater or less extent, at some point or other, as the distance is varied. They will, however, as already observed, be most prominent under circumstances the *most favourable to the inductive influence*: and it is hence not surprising that the law of action should appear by experiment to be sometimes in an inverse ratio of the distance; at others in an inverse ratio of the squares of the distances, and in others not conformable to either; whilst in some instances we observe the singular phenomena of attraction at one distance and repulsion at another.

19. One condition favourable to the disturbances above mentioned, and which it is important to notice, is the inequality of the repelling bodies in respect of extension, an increase of extension being generally accompanied by an increased inductive susceptibility. Thus it may be observed, that in connecting an insulated sphere, S, fig. 9, or other body, with the fixed ball of the balance, we give it so much inductive power that it is only in very few cases we obtain a repulsive force varying as $\frac{1}{d^2}$, notwithstanding the intensity of the opposed discs is considerable: in this case the results will be often irregular, and the respective forces very frequently as 3 : 1; when the distances are as 2 : 1, in a great variety of cases the force will be found to vary as $\frac{1}{d}$.

Experiment F.—An experimental illustration of this may be seen in the following Table. In these experiments the fixed ball *p*, fig. 3, was connected with an insulated sphere of three inches diameter; the respective forces being the result of different charges, and being obtained by means of the micrometer circle, according to the method explained in section 12.

TABLE V.

Distance.	Force.	Distance.	Force.	Distance.	Force.	Distance.	Force.	Distance.	Force.	Distance.	Force.
4	18	4·5	110	10	65	13	78	17·5	117	20	150
8	8+	9	54	20	20	26	26	35	35	26	90
....	....	18	18	....	....	....	....	....	....	40	40
....	....	....	....	....	....	....	....	....	....	52	23+
A		B		C		D		E		F	

We here perceive, in accordance with the preceding phenomena, that in column A, where the intensities are not considerable, and the distances small, the force is as $\frac{1}{d}$, or very nearly so; whereas in column F, in which the intensities are considerable, and the distances great, we have the force nearly as $\frac{1}{d^2}$. In the other columns, B, C, D, E, the forces are nearly as 3 : 1 when the distances are as 2 : 1, except in one case in

column B, where the force is again nearly as $\frac{1}{d}$ in the case of the distances being small, as at 9° and 4°·5. We also perceive in one instance, that where the distances are as 4 : 1, the forces are as 6 : 1, as in the distances 18° and 4°·5 of column B.

20. These facts are of great consequence to every species of electrical inquiry depending on any instrument whose indications are derived from repulsion. COULOMBE found, for example, that the balls of his balance were repelled with only one half the force at a given distance when the quantity of electricity in one of them was reduced to one half, and further concludes that the whole repulsive force expressed by $\frac{F}{D^2}$ diminishes for the same distance, D, as the absolute quantity of electricity in each of the repelling bodies considered as points. This principle he applied extensively, with a view of detecting the ratios of the quantities of electricity accumulated in charged bodies or in any given point of them. The electricity of the given point he considered as transferable to a small insulated disc, first applied to the body, and subsequently placed in his balance, the ball of the needle being already charged with a certain quantity of the same electricity. The insulated disc has been termed a proof plane: when this plane is placed upon any part of a charged body, it is supposed to be identical with an element of the surface, so far as relates to the distribution of the accumulated electricity; and hence, on removing it to the balance, it is assumed to operate just as the element would do under similar circumstances. Admitting, however, this identity, the indications of the instrument may not in all cases be directly proportionate to the quantity of electricity in the proof plane, since on the principles just explained, and as seen in Table IV., the respective quantities of electricity are not always as the repulsive forces: if the particular cases in which this happens be not first ascertained, we may possibly arrive at erroneous conclusions. It appears, for instance, from COULOMBE's researches with the proof plane, that the relative electrical capacities of a solid or hollow sphere and a circular plane of equal area, each side to each side, are as 2 : 1; that when electricity is accumulated on a globe, either hollow or solid, it is only found upon the exterior surface; hence in expanding the globe into a plane circular area of the same superficial extent, each side to each side, we double its capacity by giving it another exterior surface*; twice the quantity of electricity may therefore now be placed on it under the same intensity.

21. This point is of some importance to an exact electrical theory, and deserves attention. The experimental evidence in support of it is principally the following.

1°. When a small insulated body is plunged within a hollow sphere charged with electricity, it does not, on being again withdrawn, exhibit any electrical indication; whereas on touching the exterior surface, the insulated body becomes vigorously electrified.

2°. COULOMBE found, by means of his proof plane, that when a charged sphere had

* BIOT, Traité de Physique, tom. ii. p. 275.

been touched with an insulated circular plate, one of whose surfaces was equal to the exterior surface of the sphere, it exhibited only one third the reactive force which it evinced before the contact, and hence concluded that the quantity remaining on the globe after the contact, was only one third of the previous quantity; that consequently the charge had divided itself between the plate and sphere in the proportion of their assumed surfaces of action.

22. It may not be unimportant to examine the claims of this evidence, since it has necessarily considerable influence on the future progress of electricity.

In the detail of COULOMBE's experiments, given in the Traité de Physique of BIOT, the result of the contact of the plane and sphere does not appear to have been compared with any result of contact with a similar sphere; that is to say, with the electrical reaction of the original sphere when an equal sphere was substituted for the plate. According to his views, the reaction after *one* contact of the plate should equal the reaction after contact with a similar sphere, whose exterior surface was equal to the *two* surfaces of the plate. This experiment, after what has been just stated, is essential to an accurate result; since it is possible, that although the electrical reactions shown by the balance of repulsion may be nearly as 3 : 1, yet still the actual quantities of electricity may be only as 2 : 1 (15.). Not finding in any of the accounts of COULOMBE's inquiries which have come under my notice an experiment of this kind, it may be worth while to commence with this test of the theory. For this purpose I obtained two conducting spheres, each four inches in diameter, and a circular plate of eight inches in diameter, as represented by S, *s*, P, fig. 9.

Experiment G.—Having insulated and charged one of the spheres, S, as also the disc of the needle, with the same electricity, according to the method of experimenting adopted by COULOMBE, I proceeded to ascertain its electrical reaction by means of a tangent disc, and found the needle repelled 22°, the reactive force of the instrument being about the $\frac{1}{2000}$dth of a grain for each degree. The charged sphere was now touched with the insulated plate P, and its electrical reaction again observed. This being effected, I replaced the original charge on the sphere, so as to again obtain a force of 22°, and then repeated the experiment with the second insulated sphere *s*. The results are given in the following Table, in which it may be seen that the electrical reactions, after the respective contacts with the plate and sphere, the areas of which were equal; instead of being as 2 : 1, according to the theory, are nearly the same, whilst the subsequent forces at 22° distance, as compared with the reaction of the original charge of 22° at the same distance, are nearly as 3 : 1. This ratio we have found in several instances where the charges on the repelling discs are unequal and in the ratio of 2 : 1.

TABLE VI.

Reaction before contact at 22° distance.	Reactions after contact.			
	Reaction at 14°.		Reaction at 22°.	
	Plate.	Sphere.	Plate.	Sphere.
22°	14°—	14°+	7°	8°—
A	B		C	

In column A we have the force communicated to the proof plate at first. In column B the force after the respective contacts of the sphere and plate, as shown by the mere deflection of the needle. Column C shows the reaction at the original distance of 22°.

23. In order to obtain more readily a repetition of the original charge, I placed the insulated sphere immediately under the suspended plate of the electrometer described in my former paper *, and as represented for the cylindrical conductor in fig. 18. Having reproduced a given force, as indicated by this instrument, by repeatedly touching the sphere with a small transfer conductor charged to a sufficient intensity from the knob of a charged jar, the sphere was removed, and submitted to experiment. By this process the original electrical reaction of 22° could be easily obtained.

24. The result, therefore, arrived at by COULOMBE's method of experiment may be classed with those cases in which the repulsive force exercised by the balance is not proportionate to the quantity of electricity. Thus, if we suppose, by way of further illustration, that in certain experiments similar to those detailed by COULOMBE, the respective quantities of electricity communicated to the tangent disc had been really in the ratio of 2 : 1, but so circumstanced as to have been at first equal to and subsequently half the quantity with which the disc of the needle was charged; and suppose that the respective reactions had been taken at a distance of 24°, 18°, 12°, or 9°, with intensities corresponding to those given in columns B and *d* of Table IV., we might have then found the reactions in the ratio of 3 : 1, or nearly so; or supposing the proof plate, after both contacts, to have become charged to a *higher* intensity than the disc or ball of the needle, the respective quantities on the repelling bodies being in the proportions of $1 : \frac{1}{4}$ and $\frac{1}{2} : \frac{1}{4}$; that is, supposing the proof plane to have received, before the contact with the plate, *four times* as much electricity as existed on the ball of the needle, and after the contact only *twice* as much, then if the reactions were taken at a distance admitting of a sensible inductive disturbance (15.), the repulsive forces might still be in the ratio of 3 : 1, or nearly so, although the respective quantities of electricity on the proof plate producing the repulsion in each case should be really as 2 : 1. This is shown in columns *b* and *d* of Table IV., in which the forces corresponding to the distances 12°, 18°, and 24°, are nearly as 3 : 1, whilst the

* Philosophical Transactions for 1834, Part II. p. 215.

respective quantities are as 2 : 1. I have adverted to the results in Table IV. merely by way of illustration: it is however quite evident that with other intensities the coincidences alluded to might be more perfect, especially if, as in the experiment cited by M. BIOT, we had, instead of a small disc, employed an insulated sphere of an inch in diameter for testing the respective electrical reactions: in this case, as I have already endeavoured to show (19.), the disturbing force of induction would be more powerful, one of the repelling bodies having greater extension (Table V.).

25. If, instead of previously charging the disc of the needle with the same electricity, it remains neutral, and the electricity taken up by the proof plane be equally distributed between them, we may frequently avoid the disturbances above mentioned, more especially if the electrical intensities and the distances are within certain limits. Thus we observe, in again referring to Table IV., that in columns A and B, where the quantities on each disc are as 2 : 1, the forces are in almost every instance as 4 : 1, the force being as the square of the quantity; a result which I obtained also from the attractive forces by another and very different kind of experiment*.

Experiment H.—With a view, therefore, of further verifying the preceding results, the former experiments, Table VI., were repeated in this way; that is to say, the electricity was distributed equally on the two discs and the square roots of the forces taken to designate the respective quantities: under these circumstances I found the electrical reactions of the charged sphere before and after contact both with the circular plate and with an equal sphere, in the ratio of 4 : 1; hence the quantity abstracted by the plate was equal to the quantity abstracted by the sphere, and just half the original quantity. I took in these experiments an electrical reaction of 16° distance as a unit of charge; having previously ascertained by experiment that with this intensity the forces varied as the square of the quantity, or very nearly so, when the quantity on the discs was reduced to one half; the reactive force of the instrument being about the $\frac{1}{2000}$dth of a grain for each degree, and the proof plate of an inductive capacity adequate to the purposes of the experiment (42.).

26. Although the above result seems sufficiently conclusive of the point under consideration, and is quite in keeping with the result arrived at in my former paper, viz. that the capacity of a sphere is the same as that of a circular plane of equal area into which we may suppose it to be expanded†: it may still not be altogether useless to verify it by another kind of experiment, equally conclusive and direct with the preceding.

Experiment I.—The discs $p\ m$ of the balance, figs. 3 and 14, being brought into contact, and one of the insulated spheres above mentioned, S, fig. 9, connected with the fixed disc p, a charge was conveyed to it, which, on easing away the lever confining the needle, amounted to 48° at 48° distance. This being noted, the sphere S was touched with the insulated circular plate P, and the new position of the index from zero noted; the disc of the needle was then touched with a similar neutral disc, so

* Philosophical Transactions for 1834, p. 219.

† Ibid. p. 235.

as to reduce the quantity contained on it to one half, and bring it to equality with the fixed disc, on the supposition that the plate had abstracted from the sphere one half the charge. The new distance of the index from zero was again observed, and the two discs again brought into contact, so as to equalize more completely the distribution between the discs, should any difference exist. Little difference, however, was apparent; hence the assumption that the plate had abstracted one half the electricity, was to a certain extent confirmed. Finally, the electrical reactions were observed at the original distance of 48°.

This process was repeated in substituting a similar sphere for the circular plate, and also a solid sphere of the same diameter. The following results were obtained: $f_1 f_2 f_3$ designate the reactions above mentioned, the electrical reaction at the original distance of 48° being denoted by f_3.

TABLE VII.

Reactive force of the instrument $\frac{1}{2000}$.

Reaction at 48° distance before contact.	Reactions after contact of plate.			Reactions after contact of sphere.			Reactions after contact of solid sphere.		
	f_1.	f_2.	f_3.	f_1.	f_2.	f_3.	f_1.	f_2.	f_3.
48°	39	22·5	12	38	23	12+	39	23	12
N.	a	b	M	a'	b'	M$'$	a''	b''	M$''$

27. It may be observed in this Table, that under whatever corresponding circumstances we compare the results after contact with the plate and spheres, whether previously to equalizing the electrical state of the discs, as in f_1, or subsequently, as in f_2, or otherwise after the equalization, as at f_3, the result is very nearly the same. I repeated these inquiries with the electrometer represented in fig. 18*, and found by the attractive force of a unit of charge on the suspended neutral plane, that whether an electrified sphere was subjected to the contact of a circular plate of equal area, or otherwise to the contact of a similar sphere, hollow or solid, the subsequent attractive forces were equal, and the quantities abstracted precisely one half the original quantity with which the first sphere was charged, or very nearly so, taking the square roots of the forces to represent the respective quantities. I found also in connecting the plate and sphere successively in any point with the fixed ball of the balance, and communicating to each the same quantity of electricity by means of a transfer plate, charged to a given intensity, that the electrical reactions were the same, as already shown (by another method of experiment) in my former paper†.

28. *Experiment* K.—This kind of experiment I further extended to cylinders, hexagonal and other prisms, and bodies of other forms; and find, as in the cases given

* Philosophical Transactions for 1834, p. 215. † Ibid. pp. 218, 232.

in my former inquiries*, that their electrical reactions or capacities are precisely the same as those of the plane areas, into which we may suppose them to be expanded. Thus the electrical capacity of the hollow cylinder A', fig. 19, open at both ends, is precisely the same as if it were cut through in the line *a b*, and expanded into a plane A, fig. 20, and conversely the capacity of the plate A, of inconsiderable thickness, is the same as that of an open cylinder A', fig. 19, or A'', fig. 21, into which we may suppose it to be turned without doubling over the edges, whether rolled in the direction of its length *m n* or breadth *n o*; now such could not possibly happen if their capacities were not the same, since the same quantity expanded on a double surface has a very different electrical reaction.

29. In comparing the capacities of a sphere and plate of equal area by the method just given (28.), we may place any charged body in connexion with the fixed ball of the balance,—such as a common cylindrical conductor,—since we have merely to discover the respective quantities abstracted. In every case of this kind the effect of contact with a plate and sphere of equal area will be found the same. We may also reverse the former experiment, and connect the plate with the fixed disc instead of one of the spheres, and so examine the decreased intensity of the plate after contact with the sphere or with a similar plate.

On referring to columns N; *b*, M; *b'*, M', &c. of Table VII., we observe one of those cases in which the force is nearly in an inverse ratio of the distance, the distance 23 and 48 being as 1 : 2 nearly, whilst the corresponding forces 23 and 12 are nearly as 2 : 1.

30. This simple induction of facts appears sufficiently conclusive; it clearly shows that a spherical conductor, either hollow or solid, and a plate of equal area, have the same electrical capacity, and that COULOMBE's experiments are not opposed to such a conclusion.

31. We may now proceed to consider the case of an insulated body plunged within an electrified sphere, and this will necessarily lead to some further inquiries into the action of the proof plane, and to the conditions under which one substance receives electricity from another.

The curious fact that we do not abstract any portion of the charge accumulated on a hollow sphere by touching its interior surface with an insulated neutral disc placed wholly within it, will, on inquiry, be found little conclusive of the non-existence of electricity upon that surface; it may be experimentally shown that any insulated body plunged within an electrified shell, could not possibly take up a particle of electricity, even although a powerful accumulation *actually existed there*.

Experiment L.—Let a small sphere of glass *a a'*, fig. 22, made clean and dry, having a projecting neck at *a'* varnished with shell lac, be nearly filled with dry mercury, and let the whole be placed in a vessel *b b'*, also containing mercury, so as to give the glass an outer and inner coating; charge this system, and remove the charging

* Philosophical Transactions for 1834, pp. 232, 233.

wire d by an insulating handle $n\ d$, and subsequently the charged sphere $a\ a'$. Let the mercury contained within the sphere be now poured off; we have then a spherical body, upon the interior surface of which there is a powerful accumulation of free electricity. Insulate this charged sphere, and touch its interior surface with an insulated proof plate. The plate will not, on being again withdrawn, exhibit the least electrical indication, although electricity in a free state is *known* to exist upon its inner surface. If, however, we connect the proof plane with an insulated conducting rod projecting beyond the sphere, electricity will be freely taken up, as in the case of a body similarly circumstanced, and placed within a charged sphere of metal.

32. It is of no consequence to this experiment whether the glass sphere have subsequently to charging an external coating or not. Free electricity is everywhere diffused on its interior surface, and is easily communicable to any body capable of receiving it. The following experiment is sufficiently illustrative of this.

Experiment M.—Let a circular plate of glass $d\ d'$, fig. 23, be placed on a conducting plate c, whose diameter is about half that of the glass; place a similar plate c' upon its upper surface, immediately opposite the conducting plate c below, so as to give the glass two moveable coatings; charge this system by communicating electricity to the upper plate; remove the coatings, and place the glass on an insulating support; if the charged side be now touched with the proof plane, electricity will be freely taken up.

33. We have here a direct experimental fact, showing that the neutral state of the proof plate is by no means evidence of the non-existence of electricity upon the interior surface of a charged sphere, and therefore no influence of this kind can be logically deduced from the experiment in question (21.). Moreover, the theory itself assumes, upon certain principles in the 12th section of NEWTON, that the action of a spherical stratum of electricity upon any point placed within it is equal to zero: should electricity therefore *really* exist upon the interior surface of the sphere, it could not, if the theory be true, be imparted to a body placed wholly within it. The experiment therefore is in this point of view irrelevant; but if we take the experiment as a mere fact abstracted from hypotheses, it must necessarily be considered in connexion with other facts, such as those above stated (31.) (32.); in either case, however, it is clearly no evidence of the non-existence of electricity upon the interior surface of a charged shell.

34. The experiments last mentioned (L.), (M.) are instructive; they show that it is not only from the absence of electricity upon the interior surface of a charged shell that we fail to electrify a small insulated body placed within it, but that this result may also arise from incapacity of the given body itself to take up electricity under certain circumstances. Pursuing therefore these facts, unfettered by any hypothetical view of electricity, we are immediately led to examine under what conditions one body can receive electricity by communication from another, as in the case of touching a charged conductor with a small insulated disc of inconsiderable thickness; since

upon the indications of this instrument, termed a proof plane, all the experimental evidence of the theory of electrical distribution in charged bodies of various forms depends.

35. Without embarrassing the inquiry by any theoretical disquisition, let us study the phenomena as they immediately present themselves. There exists in electricity, as already observed, a peculiar kind of influence termed induction, not recognised in other invisible natural agencies, magnetism excepted; that is to say, the attractive force displayed by these wonderful powers is invariably accompanied by a previous change in the electrical or magnetic states of the attracting bodies. If, as in the case of electrical attraction, the forces thus generated, as it were, by induction, be sufficiently powerful to overcome all impediment to the free communication of electricity, these induced states are found to vanish, and a new disposition of the accumulation immediately takes place, and it is thus one body receives electricity from another. I hope at an early period to lay before the Royal Society some new phenomena in electricity calculated to throw further light on the operations of electrical induction and attraction, and from which it would appear that the attractive force between a charged and insulated body in a neutral state, is entirely dependent on the reciprocal inductive forces of which, under the given circumstances, both the bodies are susceptible, and not necessarily on the mere quantity existing on the charged body. The free electricity therefore which a small proof plane takes up from a charged body may not only depend on the quantity actually existing in any given point to which it is applied, but on the reciprocal inductive force of which the bodies are susceptible at such point of application. Should the inductive capacity of the proof plane become in any way affected by position, or by its thickness, or extension of any kind, or should the charged substance be itself more or less susceptible of induction in different points, then the inductive forces will be different, and the attraction generated between the charged body and the proof plane not always proportionate to the actual quantity of electricity present, as can be shown by many striking experiments. The proof plane would not under these circumstances take up in every situation a quantity of electricity proportionate to that of the element of the surface to which it is applied. That something of this kind takes place, may, I think, be made evident in the following way.

Experiment N.—Take three equal and similar circular metallic plates, p, p', p'', figs. 24, 25, 26. Let two of them, p', p'', be hollowed up into shallow spherical segments; insulate these bodies in the positions shown in the figures, the convexity of the segment p' being placed uppermost, and that of p'' downward; charge these insulated conductors with the same quantity of electricity, which may be readily effected by the methods explained in my former paper*. Now the respective intensities, as measured by the connexion of the charged bodies with any electrometer, are all equal† (28.); hence there cannot possibly be any exception taken on account of a supposed double sur-

* Philosophical Transactions for 1834, p. 218.

† Ibid. p. 233.

face of action, as respects the plate p; for, as already observed, if such really existed, the plate could not possibly evince the same intensity with the same quantity, as is shown by disposing the electricity on two plates instead of one, in which case the attractive force, as evinced by the electrometer, is in an inverse ratio of the square of the surface*; a result also demonstrable, in many instances, by means of the repulsive force communicated to the discs of the balance, fig. 3. Under these circumstances it is not illogical to conclude, that so far as surface is concerned, the electricity is upon the whole circumstanced much in the same way in each. Let a small proof plane be now applied to these charged bodies successively, in the similar points p, p', p'', and the respective electrical reactions observed. The greatest electrical reaction will be obtained on the convexity p', the next on the plane p, and the least on the concavity p''.

If in these experiments p, p', p'' represent the situation of the proof plane, we observe that in the concavity p'' it is most completely enveloped in the electrical molecules of the charged body; on the plane it is much less enveloped; on the convexity of the segment p' the particles of electricity are bent away from it, as it were, in all directions. Position alone therefore in respect of the other parts of the charged body may possibly influence the quantity taken up by the insulated plane. Now whatever tends to increase the inductive susceptibility of the proof plane, will bring the electrical reactions nearer a ratio of equality. Thus in giving the proof plane some considerable extension in the direction of its thickness, or otherwise in holding it by an insulated metallic wire, w, fig. 27, we increase its inductive force; in this case the differences in the electrical reactions with a given charge become less.

36. It follows from this, that could we employ a proof plane of perfect inductive susceptibility, we should actually arrive at equality in the reactions of the three bodies above mentioned. One method of effecting this is to make the proof plate a portion of a small coated element of glass, as represented in figs. 5 and 8. If a compound element such as this, and which has been already described (8, δ.), be substituted for the small insulated disc above mentioned, it will, after contact with the charged bodies p, p', p'', fig. 24, have an equal reaction imparted by each. Another method consists in extending the limits of the wire w, fig. 27; but this we do in connecting the bodies with the fixed disc of the balance in the way above mentioned (8, ν.); and we accordingly find that the repulsive forces imparted to the discs of the balance are equal.

37. *Experiment* O.—Should any doubts remain concerning the actual disposition of the electrical charge on the three bodies p, p', p'', figs. 24, 25, &c., we may substitute coated glass for these bodies, and charge them each equally by means of a small unit jar†. We have then, on removing the coatings from the charged side, or otherwise both the coatings, three strata of free electricity of the forms above mentioned. If any difficulty occur in obtaining two spherical segments of glass sufficiently similar, we may charge the opposite sides of the same segment successively. The temporary

* Philosophical Transactions for 1834, p. 219.

† Ibid. p. 217.

coatings both of the plate and segments may consist of sulphate of lime, neatly moulded to the glass, and subsequently gilded; they must be each of an equal surface. It will be also requisite for greater accuracy to obtain a glass plate of the same thickness as the spherical segment.

It is not therefore impossible for a proof plane, such as that employed by COULOMBE, to exhibit unequal electrical reactions, and yet the distribution on the conductor to which it has been applied be uniform. Thus a tangent plane of inconsiderable thickness applied at the extremity a of a charged cylindrical conductor, c, fig. 18, may be in a more favourable position for the inductive action than if placed at the centre c, or in the centre of the circular plane c', terminating either of its extremities.

38. In treating of the proof plane, philosophers have considered its action in more than one point of view. M. BIOT states that the proof plane, in becoming assimilated with a superficial element of a charged body, will take up as much electricity upon each of its surfaces as exists upon the point to which it is applied; hence, on removal, it is charged with double that quantity*. M. POUILLET, on the contrary†, considers the proof plane to be, at the instant of contact, in precisely the same state as a superficial element of the same dimensions, and to be, on removal under the same electrical conditions, as a similar portion of the charged body would be placed if actually cut out of its surface; that is to say, according to his view, the electricity would be first collected on one side only, and would subsequently be expanded over both; each surface therefore has only half the quantity which the superficial element at first possessed.

39. Both these views of the state of the proof plane are evidently at variance with the phenomena above recorded (28.); this, it is true, is of no great consequence to COULOMBE's results when he merely employs the proof plane to determine the ratios of the quantities of electricity distributed on a charged conductor; it has, however, still a material influence on the theory of electricity.

40. The experiments with the proof plane just mentioned (31.), (32.), together with the phenomena of repulsion so frequently alluded to in this paper (14.), necessarily lead us to investigate more rigorously the nature of its indications, in order to discover, if possible, the conditions under which it may fail to become charged, either with the same quantity as exists in the points touched, or otherwise in the ratio of the quantities.

The first notions which present themselves from our experience of ordinary electrical actions, would lead us to conclude that an electrified conductor of large dimensions could always charge a small body to saturation from any point of it, provided the electrical state of the touching body was such as would enable it to become so charged: this is indeed quite evident from the fact that a compound element su chas that already mentioned (8, δ.), and represented fig. 8, becomes charged equally at whatever point of a long electrified cylinder, c, fig. 18, it is applied, whether

* Traité de Physique, tom. ii. p. 271. † Physique Expérimentale, tom. i. Seconde Partie, p. 579.

at the centre c or at the extremity a; as also from the fact already noticed (36.), that whatever part of a charged conductor be connected with the fixed disc of the balance, or with any other species of electrometer, the instrument is equally affected. The proof plane, therefore, if it really exhibits the ratios of the quantities of electricity disposed in different points of a charged body, must depend materially on the resistance to an equal participation in the charge, arising from its little inductive susceptibility; it is hence very desirable to investigate experimentally the various effects which increased thickness, or extension of any kind, position, intensity of charge, and the like, may have on its indications. An experimental inquiry of this kind is, however, extremely difficult: we are, for example, open to all the sources of fallacy above described (15.), and illustrated by Table IV. Thus, if we examine the electrical reactions, having previously charged the disc of the needle with some given quantity of the same electricity, the subsequent quantities taken up by proof planes of various thicknesses, &c., may bear all sorts of proportions to this quantity; and since the law of action is under these circumstances not always regular (15.), we might have, in examining the respective forces at some given distance, d, very obscure and uncertain results. If, on the other hand, we neutralize the disc of the needle at each experiment, and diffuse the electricity taken up by the proof plane equally over each, we are still open to fallacy; since at small distances the force is sometimes in an inverse ratio of the simple distance, whilst at others it varies in an inverse ratio of the square of the distance (17.); and we have in some instances the electrical reaction so little that the discs do not separate, in consequence of the slight attractive force exerted between them at the point of contact, and which in some degree vitiates the result at a given distance, d.

41. *Experiment* P.—Notwithstanding these obstacles, I have endeavoured to obtain a good series of observations on the indications of insulated tangent planes of various thicknesses, applied to different points of a charged cylinder of about four feet in length, and 2·5 inches in diameter, terminating in plane circular faces. The experiments were conducted in the following way. The charged cylinder c, fig. 18, was placed on two slight insulators, I, I′, fixed on a small platform, N, supported beneath on four small rollers, so as to be moveable between the guide pieces g' g' of the fixed base B. The extremity of the cylinder could be by this method brought immediately under the suspended plane p of the electrometer E*. When it was required to convey to the cylinder a charge of any given magnitude, the index of the instrument was adjusted from zero to a given number of degrees in the direction $o\ y$, by means of small weights placed in the cup at q. Electricity was then communicated to the cylinder c until the index came again to zero; and thus the distance $p\ a$, at which the attractive force operated on the suspended plane p, was always constant for any given charge. Now as the attractive force is as the square of the quantity of electricity on the charged body†, we have only to make the degrees at which we adjust the

* Philosophical Transactions for 1834, p. 215. † Ibid. p. 221.

index from zero vary in this ratio, and we obtain a double, treble, &c., quantity on the conductor, when the index is again brought to zero of the arc $x\ y$; a result which I further verified by actually placing the required quantities on the cylinder*; these methods were thus brought to check each other. Immediately the index was stationary at o; the charged cylinder was withdrawn from the electrometer, so as to avoid any possible determination of the charge toward the extremity, and the tangent plane immediately applied and examined by the method before mentioned (ν.), (8.). The cylinder was, at each successive experiment, again brought under the electrometer plane, and the electricity which had dissipated during the last experiment replaced; the tangent bodies employed, all exposed circular touching planes of half an inch in diameter; their extensions in the direction of the thickness were as follows: taking the letters $a\ b\ c$, &c., to represent the plates, and the unit of length $= 1$ inch, then $a = \cdot005$, $b = \cdot12$, $c = \cdot25$, $d = \cdot5$, $e = 1$, $g = 2$.

In addition to these, I tried a tangent plane backed by a wire of four inches in length, as represented in fig. 27, and also a compound element of coated glass $= q$, fig. 8, the coatings being areas of the same dimensions as the others, viz. ·4 of an inch in diameter, and of inconsiderable thickness. In observing these reactions the bodies were transferred to the balance, the disc of which was previously neutralized, and the electricity equally disposed upon the repelling bodies. The first deviation of the needle was then observed $= f$, and finally the force taken at $10° = F$, considered as a unit of distance. The square roots of the forces were then taken to designate the respective quantities of electricity, or the ratios of the quantities existing on the different points of the charged cylinder. The points of the cylinder C touched, were the centre E, the extreme end C', and the centre of the plane face terminating its extremity C'. Little difference being discoverable in the points between the centre and extremity, I have condensed the general results of this investigation within as short a space as possible: they are as follows. The letters $a\ b\ c$, &c., denote the different tangent bodies, the reactive force of the instrument being about the $\frac{1}{2000}$dth of a grain for each degree; f represents the first deflections, as also the distances of the repelling bodies; F the reactions taken at a given distance; $d = 10°$; C C' E denote the points touched.

Table VIII.

	Quantity of electricity $= 1 = 32°$ of electrometer.															
	a.		*b.*		*c.*		*d.*		*e.*		*g.*		*p.*		*q.*	
	f.	F.	*f.*	F.	*f.*	F.	*f.*	F.	*f.*	F.	*f.*	F.	*f.*	F.	*f.*	F.
C..	3+	1—	5	2·5	6·5	4	10	10	14	25	20	55	28	168	32	328
C'..	4·5	2	7+	5	9	8	14	25	20	55	29	220	31	320	32	328
E..	6	3·5	10	10	14	26	19	50	23	92	30	225	32	323	32	328

* Philosophical Transactions for 1834, p. 218.

TABLE IX.

	Quantity of electricity $= \frac{1}{2} =$ 8° of electrometer.									
	d.		*e.*		*g.*		*p.*		*q.*	
	f.	F.	*f.*	F.	*f.*	F.	*f.*	F.	*f.*	F.
C . .	7	4	10	10	14	25	15	25	21	81
C' . .	9	8	15	32	20	55	22	70	21	81
E . .	14	25	16	36	20	55	22	70	21	81

Supposing the given distance $d = 10°$ to be one of those (14.) which would admit of the repulsive forces being considered proportionate to the quantities of electricity in the respective points of the body touched, we have, in taking the square roots of the respective forces, at 10°, the following results.

TABLE X.

	Quantity of electricity = 1 = 32° of electr.							
	a.	*b.*	*c.*	*d.*	*e.*	*g.*	*p.*	*q.*
C . .	1—	1·6	2	3+	5	7·4	13	18
C' . .	1·4	2·2	2·8	5	7·4	15	18—	18
E . .	1·8	3+	5	7+	9·5	15	18—	18

TABLE XI.

	Quantity $= \frac{1}{2} =$ 8° of electr.				
	d.	*e.*	*g.*	*p.*	*q.*
C . .	2	3+	5	5	9
C' . .	2·8	5·6	7·4	8+	9
E . .	5	6	7·4	8+	9

42. On examining these results we observe, 1°. In the horizontal column C, Table VIII., where the quantities of electricity are less considerable, the respective forces *f* F are in the first four cases, *a*, *b*, *c*, *d*, in an inverse ratio of the distances, or nearly so: thus we have for plate *b* force at 5° : force at 10° : : 5 : 2·5 : : 2 : 1. This law, however, begins at last to change, and become irregular when the quantity of electricity increases, a phenomenon already observed in Table IV. (14.). A similar result ensues in the horizontal column C', except that as the quantities of electricity are more considerable the law begins sooner to change, as at the plate *d*. The same result is seen in the column E; but here the quantity of electricity being greatly increased, the law changes still more early, as at the plate *c*. In Table IX. we observe similar effects; thus verifying in great measure the principles and conclusions we have already arrived at (14.). When, therefore, we begin to reduce these results so as to obtain the ratio of the quantities of electricity supposed to be distributed on the charged cylinder, we should expect to find them more or less disturbed by the variable inductive action between the repelling bodies. We accordingly perceive, on referring to Table X., that the ratio of the centre C to the extremity E is at first, with plates *a* and *b*, as 1 : 2, or nearly so; being a similar result to that obtained by COULOMBE *. Under plate *c* this ratio tends to change, as more clearly shown in Table VIII.: in fact, it is at this point the law for the plate *c*, touched at the extremity, begins to

* BIOT, tom. ii. p. 275.

vary, as seen in the same Table. When, however, the two columns C and E again vary together, or nearly so, the ratio of the quantities, as given by each of the discs, is about the same as before, until at last, where the induction upon the tangent plate is the most perfect, the ratio becomes one of equality, or nearly so. The same remarks apply in comparing the horizontal columns C' and E or C and C'.

43. In comparing Tables X. and XI. there is one important fact to be observed; viz. that we do not get the exact ratio of the respective quantities of electricity on the charged cylinder until we arrive at the most perfect inductions, as in the action of the plates p and q: here the reactions are as $2:1$, or nearly so, whereas in comparing the similar plates and parts touched with the less powerful plates $d\ e\ g$ the discrepancies are considerable, evidently showing that we have not really arrived at the true ratio of the quantities by means of these plates.

44. It is worthy of further inquiry, whether the proof plane be really identical with an element of the charged surface to which it is applied, or whether it be not in the condition of a neutral insulated body placed within an extremely small distance of a charged body, and subject to the same laws as subsist between two such bodies when placed under similar conditions, at more sensible distances, and at which electricity may be communicated. A rigorous examination of this question would probably elucidate many phenomena of electrical action at present involved in doubt: in the mean time it may not be unimportant to review such facts connected with this point as are already known.

45. It has been found, for example, that the attractive force between charged and insulated neutral bodies is less than when the latter are uninsulated; that perfect insulators are not sensibly attracted by electrified substances; and that, in every case of electrical attraction, the force is (as already observed,) proportionate to the previous induction of which the bodies are susceptible. In accordance with these facts a perfectly insulating disc reposes on a charged surface without becoming itself electrified; an insulated neutral conducting disc more or less so, in proportion to its thickness (42.); whilst an insulated plate whose inductive power is nearly perfect, is charged to equality with the point touched (36.).

I have found in the course of some recent inquiries that the attractive force between an electrified plane surface and an insulated disc of inconsiderable thickness, in a neutral state, is frequently in an inverse ratio of the distance between the two planes; the induction of which such a disc is susceptible being extremely limited; that on increasing the thickness the force also increases up to a certain point, where, under the given conditions of distance, quantity of electricity, and the like, the induction on the opposed surface remains nearly the same.

46. These facts, together with those already mentioned in the course of these inquiries, render it highly probable that the quantity of electricity taken up from the surface of a charged body by a small insulated disc of inconsiderable thickness may be greatly influenced by the position of the point of application, independently of the quantity of

electricity; so that the same quantity may possibly exist in two different points, and yet the proof plane become charged in a different ratio, the inductive power of the plate being different in these points. M. Biot, in the second volume of his Traité de Physique*, has given, from Coulombe's manuscripts, an account of the application of the proof plane to an electrified lamina of steel plate, eleven inches in length, one inch in width, and half a line thick. He states, that when the proof plane, which was an inch long and about a quarter of an inch wide, was applied beyond the extremity of the charged lamina, so as to touch its two opposite surfaces, the plate abstracted the electricity of both, and exhibited a reaction double of that which it showed when only applied at the extremity of one of the surfaces. Now it will be immediately perceived that in this application of the proof plane it was placed *entirely without* the charged body, the most favourable position possible for taking up the electricity of the plate; a circumstance which would greatly influence the result (35.). Should this ingenious experiment really prove the diffusion of a stratum of an invisible subtle fluid over the two surfaces of the plate, it would at the same time equally well demonstrate the existence of electricity upon the interior surface of a hollow body, as, for example, upon a hollow cylinder, into which we may suppose the steel plate in question to be formed, since we have shown (28.) that the intensities evinced by a given quantity of electricity are the same in both cases, and that consequently the distribution, so far as respects these two surfaces, must be similar.

47. Should the inductive susceptibility of the tangent disc be at any time, by its position in respect of the electrical particles, reduced to zero, or nearly so, it would then fail to become in any degree charged, and would be as inefficient as a plate of varnished glass or any other non-conductor, the inductive susceptibility of which is so little, that it will not, under ordinary circumstances, take up the least electricity on being applied to a charged body. Now it is not improbable that a small insulated plate plunged within a spherical charged shell is thus circumstanced: it may hence fail to become charged, even although electricity should really exist there, and which fact we have experimentally shown (31.). Similar effects would ensue in placing a neutral body under any other circumstances involving similar conditions, as in placing a very small conducting ball immediately between two large electrified globes. The small globe does not, under these circumstances, according to Coulombe, take up any free electricity.

48. It would be difficult, without the aid of induction, to explain in what way the mere position of a neutral body, in respect of the electrical particles by which it is surrounded, effects its power of absorbing electricity; and even with this we require a more extensive investigation of the phenomena than has yet appeared. That a change of position of the electrical particles of a charged surface in respect of each other, and of the body charged, is attended by important consequences, has been already shown by Volta, who observed the curious fact that the intensity of a charged

* p. 275.

plate becomes greatly diminished as its length is increased, although the *area* of the plate and the *quantity* of electricity remain the same; a subject which I have already treated of to some extent *. But in extending a plate in length, we elongate, as it were, the stratum of electricity resident about it, and thus place the electrical molecules, if such exist, in a new relation of position in respect of each other and of the general surface of the plate. If so apparently trifling a change as the extension of an electrified plate in length, the area not being diminished, is capable of diminishing the attractive force of the charge so much as to reduce it nearly in an inverse ratio of the length †, it is not unreasonable to suppose that the position of the proof plane, as respects the mass of the electrical molecules, may have an important influence on its indications.

Suppose, for example, the square plate *a d*, fig. 28, to be charged with electricity, and to a given intensity; imagine its area to consist of thirty-six equal squares, each an inch square, the side of the plate being six inches; then if we imagine the same area to become placed under the rectangle *a' d'*, thirty-six inches in length, and only one inch in width, the thirty-six small squares will, as is evident, assume another arrangement in respect of each other. Any one square will be in contact with only two others, they will have, as it were, at least two sides free, whilst those at the ends *a' d'* will have three sides free. Now in the square *a d*, each of the smaller squares is placed between four others, except those at the edges, which have one side free, and those at the angles, as at *a* and *d*, which have two sides free.

It is always difficult, in treating of so incomprehensible an agency as electricity through the medium of effects, to avoid altogether certain hypothetical analogies and forms of expression: it will be however understood that in resorting to such analogies, they are to be considered merely as philosophical contrivances, introduced for the purposes of illustration, and not in any way subservient to an exclusive theory of electrical action.

Under this necessary limitation, let us imagine the distribution of the electricity upon the square *a d*, to be, in the absence of any other conducting body within the sphere of its influence, uniform. Then, as is proved by experiment ‡, the capacity of the area *a d* is increased when it becomes placed under the rectangle *a' d'*; we can hence place a greater quantity on the plate *a' d'*, under the same intensity, than on the square *a d*. Imagine a proof plane to be applied to the square *a d* at the centre *c*, and to be in the state of any other insulated neutral body placed, under ordinary circumstances, very near it, without becoming identified with an element of the surface: then the same cause, whatever it be, which affected the capacity of the square *a d*, considered as a whole, may also affect the capacity of the proof plate considered as a whole. We may infer, for example, that at the centre *c* the electrical particles to which it is immediately applied are enveloped on all sides by other particles, and that hence none of the *sides* are free. When, however, we remove the plate to the

* Philosophical Transactions for 1834, p. 232. † Ibid. p. 233. ‡ Ibid. p. 232.

angle *a*, the particles are differently circumstanced, and have one or two sides free. Resting, therefore, on the previous fact, that the capacity of a conducting body, considered as a whole, and upon which we accumulate electricity, is affected by this circumstance, we might be led to conclude that the proof plate of small thickness should actually receive more electricity at the angle *a* than at the centre *c*, it being there less exposed to the opposing forces, whatever they be, which tend to contravene the induction. Now if we extend the thickness of the proof plane P, we place the distant points more without the influence of these forces; hence its inductive capacity becomes more perfect, so that when the thickness or other extension is sufficiently great to render the inductive capacity the same at the centre *c* as at the angle *a*, then the quantity of electricity abstracted is also the same, as is found by experiment (36.) (41.) Tables VIII. and IX.

49. It is somewhat doubtful therefore, whether we can really take the proof plane as an element of a charged body, since it forms no integral part of the surface, as is the case with the point touched. It may, however, be still open to the same influence as that which affects the capacity of the whole area to which it is applied; so that the disposition of electrified bodies to yield up their electricity at points or edges may as easily arise from the superior attractive force generated by a more perfect induction in external bodies, in the way just stated, as from an original concentration of the charge upon such points or edges. In short, we really know nothing of the actual distribution of electricity upon a charged surface, except through the medium of other bodies in some way applied to it. I have already endeavoured to show * that an electrified substance only gives off electricity by the influence of an attractive force, set up between it and some other substance: hence an electrified sphere or other body perfectly insulated in the best vacuum which can be obtained, under ordinary circumstances; will, if placed without the influence of any source of attraction, retain its electrical state for an indefinite period †. It is therefore not until we present a neutral conducting body to an insulated charged body that we begin to disturb the electrical distribution, which *may have* been previously uniform.

50. In the course of this and the preceding communication I have ventured occasionally to scrutinize the prevailing theories of electricity, and advert to the opinions entertained by many profound inquirers in this department of science. I would not, however, be thought insensible to their claims on our confidence. The researches of many distinguished philosophers on the Continent, together with those which have reflected so much honour on the science of our own country, must necessarily receive from every impartial mind the warmest admiration. It must not, therefore, be forgotten, that whilst detailing a series of facts carefully deduced by induction and ex-

* Philosophical Transactions for 1834, p. 242.

† Ibid. p. 244.

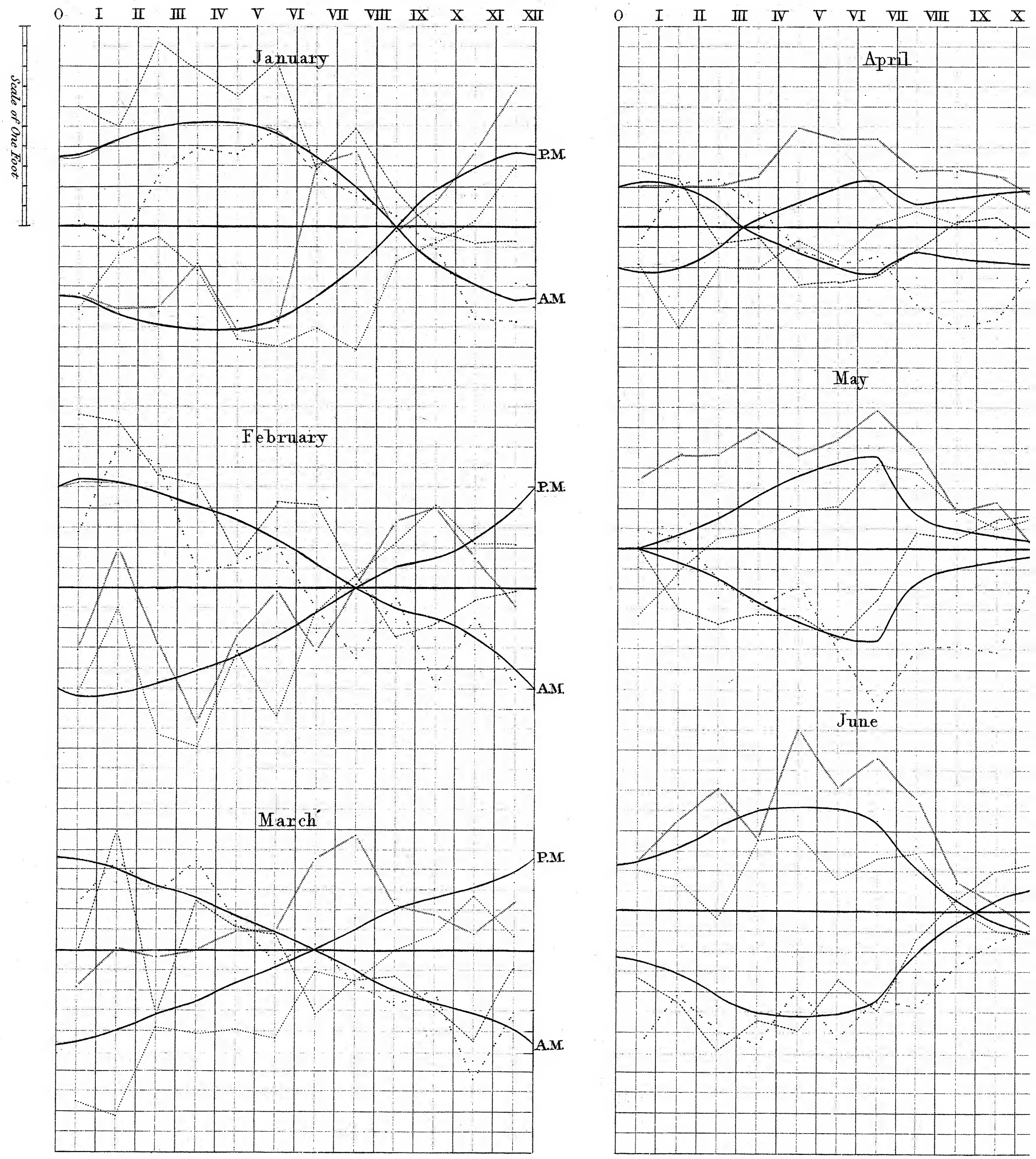
periment, I have no undue bias in favour of peculiar views of my own; my only object being, by new physical researches, to improve our acquaintance with one of the most subtle and powerful agencies in nature.

Plymouth,
April 12*th*, 1836.

Diurnal inequality in the height of high water

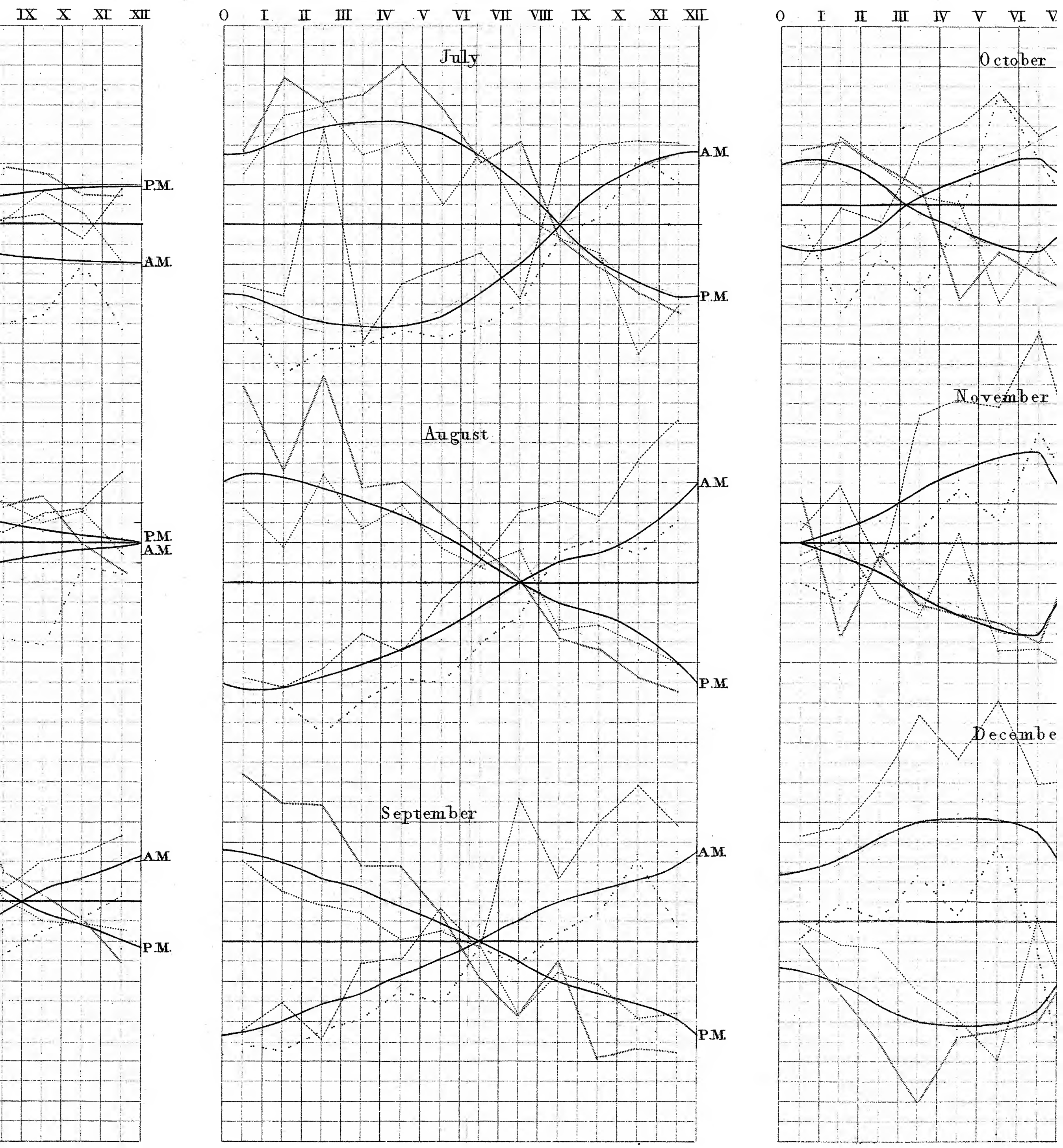
Upper Transits .... *P. M* ....
Do. .... *A. M* ....
Lower Interpolated Transits .... *A. M* ....
Do. .... *P. M* ....

The letters A.M, P.M refe
The continuous lines have b
between these lines may be



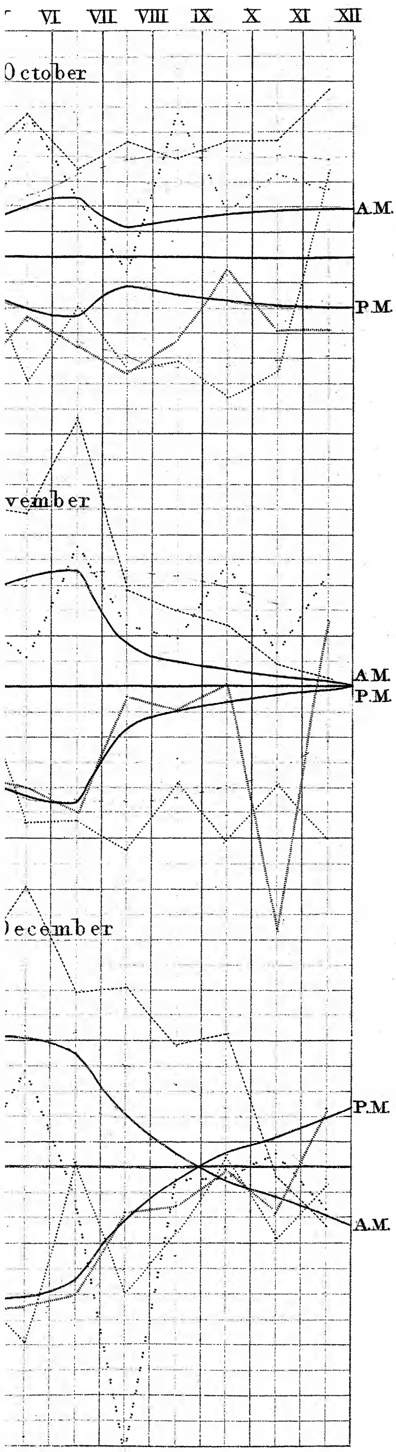
water at Liverpool from 13,327 observations. — See Tables XXVIII and XXIX.

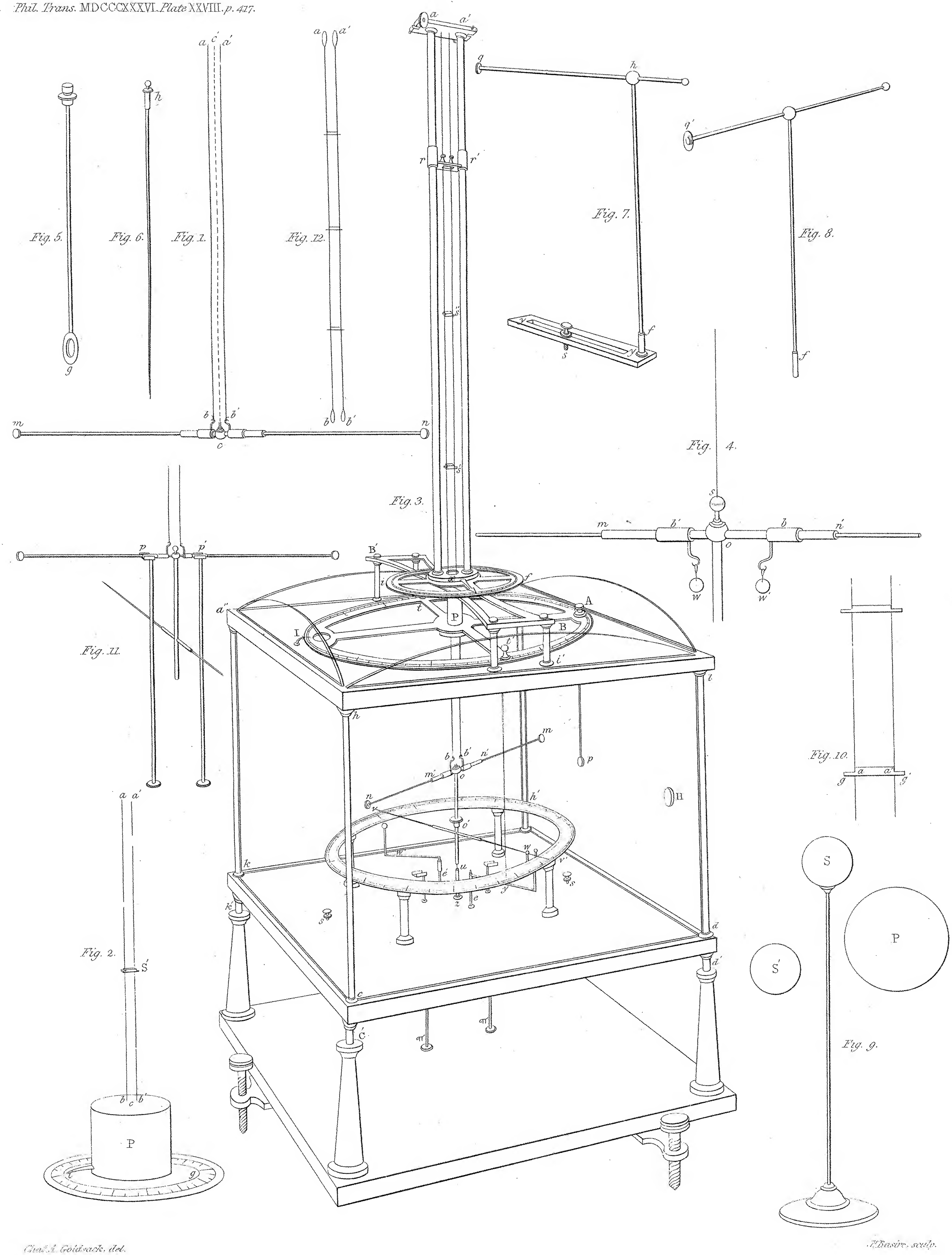
.M refer to the time of the moon's transit preceeding the time of high water.
have been laid down from Table XXIX. which is conjectural and formed by arbitrary alterations from Table XXVIII
may be considered as the difference between the height of the morning and evening Tide for the middle of each month



XXIX.

e XXVIII. *The interval*
onth





Chas. A. Goldsack, del.

J. Basire, sculp.

Diurnal inequality in the height of high water at Liverpool from 13527 observations _ See Tables XXVIII and XXIX

Upper Transits ... P. M.
Do. .... A. M.
Lower Interpolated Transits ... A. M.
Do. .... P. M.

The letters A. M. P. M. refer to the time of the moon's transit preceding the time of high water.
The continuous lines have been laid down from Table XXIX, which is conjectural and formed by arbitrary alterations from Table XXVIII. The interval between these lines may be considered as the difference between the height of the morning and evening Tide for the middle of each month.